NUMERICAL SIMULATION OF MAGNETIC RECONNECTION IN ERUPTIVE FLARES
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ABSTRACT
Prompted by the Yohkoh observations of solar flares, which have established the essential role of magnetic reconnection in the release of energy, we have studied the evolution of eruptive flares in some detail based on the reconnection model by means of the two-dimensional magnetohydrodynamic (MHD) simulations. We are interested in what factor affects the time evolution of solar flares and how the related phenomena, particularly observed loop-top source and plasmoid eruption, can be explained by this model. We have studied the dependence of the structure and evolution of the system on plasma \( \beta \) (ratio of gas pressure to magnetic pressure), adiabatic index, \( \gamma \), and \( \rho_0 \) (initial density in the current sheet). If the timescale and velocity are normalized by Alfvén time and Alfvén speed, respectively, we find that the main results (e.g., reconnection rate, plasmoid velocity, etc.) are rather insensitive to the plasma \( \beta \). The \( \rho_0 \) value, on the other hand, crucially affects the motion of a plasmoid: the ejection velocity of plasmoid grows in proportion to \( \rho_0^{-1/2} \) in the early phase, which suggests that the observed plasmoid velocity can be reproduced when we assign \( \rho_0 \sim 40 \rho_0 \) (initial density outside the current sheet). When adiabatic index \( \gamma \) is small, corresponding to the case of efficient thermal conduction, plasma heating will be generally suppressed, but the compression effect can be rather enhanced, which plays an important role in forming the high-density loop-top source. We discuss loop-top sources, plasmoid eruption, and the rise motion of a loop in comparison with the observations. Our simulations can well account for the existence of the loop-top, hard X-ray sources discovered in the impulsive flares. We concluded that both the impulsive flares and the LDE (long duration event) flares can be generally understood by the reconnection model for the cusp-type flares.

Subject headings: MHD — Sun: corona — Sun: flares — Sun: magnetic fields —
Sun: X-rays, gamma rays

1. INTRODUCTION

Since the launch on 1991 August 30, the solar X-ray satellite Yohkoh has been collecting a variety of beautiful X-ray images of the Sun, which has promoted a great deal the research toward physical understandings of solar corona. Solar flares are one of the main targets of Yohkoh. So far, in fact, many details of time-dependent, spatially well-resolved flare structure have been revealed. Through extensive Yohkoh observations, it has been established that magnetic reconnection plays an essential role in the release of energy in flares (e.g., Tsuneta & Lemen 1993). In addition, Yohkoh has discovered many kinds of related phenomena, such as X-ray jets and X-ray plasma ejections (for a review, see Shibata 1995).

There are several different ways to classify solar flares (see Table 1). Here we distinguish between the LDE (long duration event) flare and the impulsive flare. Morphologically, the former has a cusp-shaped loop structure, while the latter does not, having only a simple loop structure. Since these two types of flares are different in appearance, it has been long thought that two distinct mechanisms may be operating for each. For example, the LDE flare (hereafter referred to as cusp-type) has been understood by the classical model for two ribbon flares; namely, it was long suspected that magnetic reconnection occurs above the loop top and most energy is released outside the loop (see Carmichael 1964; Sturrock 1966; Hirayama 1974; Kopp & Pneuman 1976). Yohkoh actually confirmed this hypothesis

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2. MATHEMATICAL FORMULA AND PHYSICAL CONDITIONS

2.1. Basic Equations

We consider magnetized gas layers composed of an ideal gas. The effects of gravity are neglected for simplicity. Using the Cartesian coordinates, the basic equations are

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \]

(1)

\[ \rho \left[ \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right] = -\nabla P + \frac{1}{4\pi} (\nabla \times \mathbf{B}) \times \mathbf{B}, \]

(2)

\[ \frac{\rho'}{\gamma - 1} \left[ \frac{\partial (\rho')}{\partial t} + (\rho' \cdot \nabla) \rho' \right] = \frac{\eta}{4\pi} |\nabla \times \mathbf{B}|^2, \]

(3)

\[ \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) - \nabla \times (\eta \nabla \times \mathbf{B}), \]

(4)

\[ P = \frac{\rho \mathcal{A} T}{\mu}. \]

(5)

In addition, we use \( \nabla \cdot \mathbf{B} = 0 \) as an initial condition for equation (4). Here all the symbols, such as \( P, \rho, T, \nu, \) and \( B \), have their usual meanings, \( \gamma \) is the adiabatic index, \( \mathcal{A} \) is the gas constant, \( \mu \) is mean molecular weight, and \( \eta \) is magnetic diffusivity.

2.2. Units

All the physical quantities in equations (1)–(5) have dimensions. Next we define the dimensionless variables by introducing three characteristic quantities: the reference length, \( L_0 \), the initial sound speed, \( C_{s0} = \sqrt{(\gamma \mathcal{A} T_0 \mu)^{1/2}} \), where \( T_0 \) means the initial temperature, and density, \( \rho_0 \), outside the current sheet, respectively. The dimensionless variables (primed variables) are then

\[ L' = \frac{L}{L_0}, \quad \nu' = \frac{\nu}{C_{s0}}, \quad \rho' = \frac{\rho}{\rho_0}, \quad P' = \frac{P}{\rho_0 C_{s0}^2}, \]

\[ T' = \frac{T}{C_{s0}^2 / \gamma \mathcal{A} \mu}, \quad B' = \frac{B}{\sqrt{\rho_0 C_{s0}}}, \quad t' = \frac{t}{L_0/C_{s0}}, \]

(6)

where \( L, \nu, \rho, P, T, B, t \) are the dimensional length, velocity, density, gas pressure, temperature, magnetic field, and time, respectively; \( \eta \), the magnetic diffusivity, is also made dimensionless,

\[ \eta' = \frac{\eta}{C_{s0} L_0} = R_m^{-1}, \]

(7)

where \( R_m \) is the magnetic Reynolds number.

2.3. Initial Conditions

Initially, we assume that total pressure, \( P + (B^2/8\pi) \), is uniform in space. There is no gas motion, and magnetic fields have only the \( z \)-component. The initial conditions in dimensionless forms (after omitting primes for each quantity) are then

\[ P(x, z) = \frac{1}{\gamma \beta \cosh^2 \left( \frac{3}{2} \pi x \right)} + \frac{1}{\gamma}, \]

(8)

\[ \frac{B_z^2}{8\pi} = \frac{1}{\gamma \beta}, \]

(9)

\[ \nu = 0, \]

(10)

\[ \mathbf{B}(x, z) = -B_0 \tanh \left( \frac{3}{2} \pi x \right) \mathbf{\hat{z}}, \]

(11)

where \( B_0 \) is an initial magnetic field outside the current sheet and \( \beta \) is a parameter representing a ratio of gas pressure to magnetic pressure outside the current sheet. We study several models with different \( \beta \) and \( \gamma \) values, different initial density distribution, and different numerical resolutions, as listed in Table 2. For studying \( \beta \)-dependence, we assign \( \beta = 0.2 \) (model S; standard model), 0.5 (model B1), 1.0 (model B2), and 2.0 (model B3). Thermal conduction is not included. Instead, we calculate the low-\( \gamma \) case with \( \gamma = 1.1 \) (model G) as a limiting case of efficient thermal conduction, besides the normal-\( \gamma \) case with \( \gamma = 5/3 \) (model S).

For modeling the dense chromosphere and photosphere, we put a dense region between \( z = 0 \) and \( z = 0.1 \), in which
density is taken to be about 10 times larger than in the other region in all models. In order to see how initial density distributions affect later evolution of the system, we calculated models with four different initial density profiles in the x-direction: a model with horizontally uniform density (model D1),

$$\rho(x, z) = -4.5 \tanh \left[50(z - 0.1) \right] + 5.5 ,$$  \hspace{1cm} (12)

and models with a density concentration at the current sheet,

$$\rho(x, z) = \frac{(\rho_s - 1) \cosh^{-2} \left[3/2 \pi x \right]}{0.45 \tanh \left[100(z - 0.1) \right] + 0.55} + 1 ,$$  \hspace{1cm} (13)

where \( \rho_s \) represents the initial density in the current sheet and is \( \rho_s = 6, 11, \) and 16 for models S, D2, and D3, respectively. Temperatures are then

$$T(x, z) = \gamma \frac{\rho(x, z)}{\rho_s(x, z)} ,$$  \hspace{1cm} (14)

in all cases (see eqs. [8], [12], and [13]).

As to the number of mesh points and the mesh spacings, we distinguish between two cases, the normal case and the fine-mesh case. In the normal case, the number of mesh points is \( (N_x \times N_y) = (100 \times 100) \) and we used the uniformly distributed mesh points, \( \Delta x = 0.08 \) and \( \Delta z = 0.24 \). In the fine-mesh calculations, on the other hand, the numbers of mesh points is \( (N_x \times N_y) = (137 \times 400) \). Furthermore, the mesh points are distributed uniformly in the z-direction, \( \Delta z = 0.05 \), but nonuniformly in the x-direction to obtain a good spatial resolution in the current sheet. Accordingly, the mesh spacings in the x-direction increase from \( \Delta x = 0.02 \) in the current sheet to \( \Delta x = 0.1 \) outside the current sheet.

In summary, model S is the standard model. Models B1–B3 have higher \( \beta \) values than model S. Model D1 is a model with a uniform density (horizontally), while models S, D2, and D3 have higher \( \rho_s (> 1) \). The adiabatic index is \( \gamma = 5/3 \) except in model G, in which we take a low \( \gamma = 1.1 \). We also performed fine-mesh calculations, models H1 and H2, which have the same physical parameters as models S and G, respectively.

2.4. Boundary Conditions

Figure 1 illustrates the configuration of the simulated domain. We impose a free boundary condition at the upper boundary (at \( z = 24 \)),

$$\frac{\partial B_x}{\partial z} = \frac{\partial v_x}{\partial z} = \frac{\partial v_y}{\partial z} = \frac{\partial P}{\partial z} = \frac{\partial \rho}{\partial z} = 0 , \hspace{1cm} \nabla \cdot \mathbf{B} = 0 ,$$  \hspace{1cm} (15)

an antisymmetric boundary condition along the z-axis (at \( x = 0 \)),

$$v_x = B_z = \frac{\partial v_y}{\partial x} = \frac{\partial P}{\partial x} = \frac{\partial \rho}{\partial x} = 0 ,$$  \hspace{1cm} (16)

and symmetric conditions for the right boundary at \( x = 8 \),

$$v_x = B_z = \frac{\partial v_y}{\partial x} = \frac{\partial B_z}{\partial x} = \frac{\partial P}{\partial x} = \frac{\partial \rho}{\partial x} = 0 ,$$  \hspace{1cm} (17)

and for the lower boundary at \( z = 0 \),

$$v_z = B_x = \frac{\partial v_x}{\partial z} = \frac{\partial B_x}{\partial z} = \frac{\partial P}{\partial z} = \frac{\partial \rho}{\partial z} = 0 ,$$  \hspace{1cm} (18)

respectively. As for line tying at the base of the numerical domain, the presence of a dense layer at the base guarantees that the field lines are effectively stationary at the base. (However, some small movement of the field line at the base can be seen in Fig. 2.) Basic equations (1)–(5), together with the initial and boundary conditions, equations (8)–(18), were solved numerically using the modified Lax-Wendroff method (see, e.g., Shibata et al. 1989).

In actual calculations, we solve only the equations over a half-domain (\( x \geq 0 \)). When illustrating the results, we doubled the calculation domain, assuming a mirror symmetry at \( x = 0 \); namely, any physical quantities at \( (x < 0, z) \) are set to be equal to those at \( (x > 0, z) \) at the same time, \( t \).
anomalous resistivity can localize the diffusion region into a fairly small area, thereby yielding the "Petschek type" configuration, which is actually favored in terms of causing a fast magnetic reconnection (Yokoyama & Shibata 1994).

We assume the following form for the anomalous resistivity in the present study except that the $\eta$ is over 300, when $\eta$ is constantly set to 300:

$$\eta = \begin{cases} \eta_0 \left( \frac{v_d}{v_e} - 1 \right), & \text{for } |v_d| = \frac{j}{\rho} > |v_e|, \\ 0, & \text{otherwise}, \end{cases}$$

(19)

were $v_d$ is a relative ion-electron drift velocity, $v_e$ is a threshold velocity, and $\eta_0$ is a constant representing a magnitude of resistivity (see Ugai 1986). In the present study, we assign $v_e = 5, 7, 12, 80$ (see Table 1) and $\eta_0 = 1/15$.

3. Structure and Evolution

3.1. Evolution of System

First we overview the evolution of the system. Figure 2 depicts how magnetic reconnection proceeds in model H1 ($\beta = 0.2$, $p = 5/3$). Here the solid lines represent magnetic field lines, and the overlying arrows show the velocity. Elapsed times are $t = 0, 6, 12$, and 18 ($L_0/C_{ph}$) in the Figures 2a–2d respectively. Note that only a part of the calculated domain, $(-4, 0) \leq (x, z) \leq (4, 20)$, is displayed here.

A fast magnetic reconnection sets out at $t \sim 6$ (Fig. 2a) and a plasmoid begins to form. As magnetic reconnection still proceeds, the plasmoid moves upward, becoming larger in size. Reconnected flux flows down, and a closed-loop structure appears at $z \sim 2$ (Fig. 2c). Finally, a cup structure is formed at the top of the lower closed loop at ($z \sim 3$) after $t = 18$ (Fig. 2d). On the whole, these features are consistent with the previous results (e.g., Forbes & Priest 1982).

We have run similar models but with different $\beta$-values (models B1–B3; see Table 2). The $\beta$-dependence of the reconnection rate is depicted in Figure 3a. In this figure, we plot the time evolution of the reconnection rates for $\beta = 0.2, 0.5, 1.0$ and 2.0. The reconnection rate was derived from the electric field at the neutral point (see Forbes & Priest 1982). By using Ohm’s law ($\eta J = (c^2/4\pi)[E + (v/c) \times B]$ in cgs units), the electric field at the neutral point is proportional to $\eta J$ because $v \times B = 0$ at this point. Evidently from Figure 3a, the lower $\beta$ is, the more quickly the system evolves. Therefore, when $\beta$ is sufficiently low ($\beta \ll 0.1$), as in the solar corona, we expect reconnection to proceed more violently.

Next, we illustrate in Figure 3b the normalized reconnection rate, $\eta j / V_{A0} B_0$ (where $V_{A0}$ is the initial Alfvén velocity outside the current sheet), against the normalized time by the Alfvén transit time, $t_{A0} / V_{A0}$. It is interesting to note that Figure 3b shows similar time variation for all models with different $\beta$ (especially lower $\beta$ models, S and B1) in contrast with Figure 3a. This implies that the evolution of the system is controlled by the magnetic field; in other words, the Alfvén transit time gives a fundamental timescale for this system. Figure 4 plots the time evolution of inflow velocity of plasmas toward the neutral point and also supports the above implication. In this figure, the elapsed time is normalized by $t_{A0}$, and the inflow velocity is normalized by $V_{A0}$. Again, four models show similar behavior. Since the inflow velocity is related to the reconnection rate,
Fig. 2.—Development of magnetic reconnection in the standard model (model S). Magnetic field lines are illustrated by the solid lines, while the arrows indicate velocity fields normalized by $V_{A0}$, the Alfvén velocity outside the current sheet.
we expect that the dynamical evolution of the system is characterized by the Alfvén transit time.

We then study the dependence of the inflow velocity on $\rho_c$ by comparing model S, D1, D2, and D3. Figure 5 plots time variation of the inflow velocity normalized by $V_{A0}$ and $t_{A0}$ for different $\rho_c$. This figure shows that the inflow velocity is unaffected by changing $\rho_c$; that is, the maximum values of $V_{\text{inflow}} \sim 0.1V_{A0}$. The initial rapid increase phase is different from one case to the next because this phase is characterized not by $t_{A0}$ but by $t_{\text{inflow}} = 1/V_{A0}$, where $V_{A0}$ and $t_{A0}$ are the initial Alfvén velocity and Alfvén transit time inside the current sheet, respectively. Roughly, this phase corresponds to $6 \leq t/t_{A0} \leq 10$ for $\rho_c = 1.0$, $15 \leq t/t_{A0} \leq 25$ for $\rho_c = 6.0$, $20 \leq t/t_{A0} \leq 34$ for $\rho_c = 11.0$, and $24 \leq t/t_{A0} \leq 40$ for $\rho_c = 16.0$. From Figures 4 and 5, the inflow velocity is about 10% of $V_{A0}$ in all cases, independently of the values of $\beta$ and $\rho_c$.

3.2. Structure and Motion of Plasmoid

Let us turn to the time evolution of the plasmoid. The Figure 6a (Plate 25) displays temperature contours for model H1. It is found that high-temperature regions lie along the magnetic fields extending out from the neutral point. The highest temperature exists at the bottom of the plasmoid ($z \sim 14.5$). The accelerated plasma emerging from the neutral point collides with the bottom of the plasmoid (at $z \sim 14.5$) and forms a fast shock there. Plasma flow occurs from the neutral point upward along the magnetic field lines surrounding the plasmoid, thereby heating plasmas at its surface. (Since thermal conduction is not included in the present simulations, heat transfer takes place only by the bulk flow of hot gas. In reality, thermal conduction should play an important role in the heat transport.) In addition, slow shocks appear inside such hot regions, thus forming a triangular high-temperature area ($-1, 10$) $\leq (x, z) \leq (1, 14.5)$ with the fast shock.

Let us see, next, the density contours in Figure 6b. At first sight, this figure looks quite differently from Figure 6a. Density is maximum at the center of the plasmoid (at $z \sim 15$) and is about 10 times larger than the value outside the plasmoid. The dense matter confined in the plasmoid comes mostly from inside the current sheet. Therefore, if the plasma is initially more concentrated along the current sheet, the plasmoid will have higher density. This is of great importance when discussing the observational appearance...
of the plasmoid. Since the emissivity is more sensitive to density than temperature, the bright plasmoid in soft X-ray seems to contain high-density plasma.

As for the motion of the plasmoid, the $J \times B$ force plays a main role in accelerating it. Large upward velocities are found in the regions surrounding the core of the plasmoid. This is because large acceleration occurs due to magnetic tension force by bent magnetic fields, so that the matter inside the plasmoid can be pushed strongly upward.

The effect of $\rho_e$ on the plasmoid motion is displayed in Figures 7a and 7b. We measure the variation of the zero point (center of plasmoid) with time and calculate its velocity. In the early phase, it is well understood that the velocity of a plasmoid is proportional to that of the outflow from the neutral point; that is, $(1/2)\rho_e V_{\text{outflow}}^2 \sim B_0^2/(8\pi)$, so $V_{\text{plasmoid}} \propto V_{\text{outflow}} = V_{\text{ac}} = B_0/(4\pi \rho_e)^{1/2}$. Figure 7a shows the temporal variation of its velocity normalized by $t_{\text{ac}}$ and $V_{\text{ac}}$. All models show similar profiles in this early phase (corresponding to $0 \leq t/t_{\text{ac}} \leq 7$), when the reconnection rate increases rapidly. Therefore, we find $V_{\text{plasmoid}} \propto \rho_e^{-1/2}$ in this phase. In the late phase ($t/t_{\text{ac}} > 7$), however, differences in $V_{\text{plasmoid}}$ grow among the different models. This is because as the plasmoid grows larger in size, the matter outside the current sheet drains into the plasmoid, which weakens the effect of $\rho_e$. Instead, the plasmoid motion is affected more by the outside density, $\rho_0$. Figure 7b, whose abscissa and ordinate are normalized by $V_{\text{ac}}$ and $t_{\text{ac}}$, supports this consideration because all show similar variation after the early increase phases; that is, the plasmoid velocity eventually becomes almost constant with time, which is consistent with the results of Ugai (1995). (Differences among the terminal velocities are caused by different accelerations in the early phase.) From these results, we see that the dependence of the plasmoid velocity on the initial density changes from $\propto \rho_e^{-1/2}$ in the early phase to $\propto \rho_0^{-1/2}$ in the late phase.

Next we display the $\beta$-dependence of $V_{\text{plasmoid}}$ in Figure 8. These figures imply that $V_{\text{plasmoid}}$ normalized by $V_{\text{ac}}$ has only a weak dependence on $\beta$, $V_{\text{plasmoid}}/V_{\text{ac}} \sim 0.54\beta^{0.06}$. (In Fig. 8, the velocity is defined by the mean velocity, which is described in § 3.3.) In conclusion, the plasmoid motion (normalized by $V_{\text{ac}}$ and $t_{\text{ac}}$) depends sensitively on the initial density inside the current sheet, $\rho_e$, rather than the plasma $\beta$.

3.3. Apparent Rise Motion of Reconnected Loop

Next we turn our attention to the closed loop structure below the reconnection point. Figure 9 illustrates by the thick line how the top of the closed loop moves upward with time for model S with $\beta = 0.2$. To figure out the loop-top motion, we select first some representative magnetic field lines which were reconnected and move away from the neutral point. As time goes on, each field line moves downward, and we plot the temporal variation of its height on the $z$-axis (where $x = 0$) by the thin lines. (This downward movement appears to have been observed by Yokoh; see Hiei & Hundhausen 1996.) Then we define the place of the loop top to be the first big-bend point, which is marked with a simple asterisk on each thin line. Finally we fit these points by a curve fitting and plot the results of the fitting by the thick line. Roughly, we find that the loop top moves in proportion to $t^{0.75}$; that is, the loop rises rapidly at first but is gradually decelerated later.

Fig. 8.—The $\beta$ dependence of the inflow velocity. Roughly, $V_{\text{plasmoid}}/V_{\text{ac}} \propto \beta^{0.06}$. 

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We made the same figures for cases with \( \beta = 0.5, 1.0, \) and 2.0 and derived the mean rise velocities of the loops for each case. The mean velocity is a velocity averaged over a time span after the early phase of a rapidly increasing reconnection rate \((t/t_{A0} > 30)\). Table 3 lists the mean velocities of the inflow, upward motion of plasmoid, and rise motion of the loop. The relation between the rise velocity of the loop and inflow velocity will be discussed in § 4.

3.4. Structure of Loop

Next we examine the detailed loop structure. As was mentioned above, we calculated models G and H2 (low-\( \gamma \) models) to study the role of thermal conduction. In this sense, we may assume that the low-\( \gamma \) case roughly simulates the system in the phase at which thermal conduction works efficiently. (Precisely speaking, since the thermal conduction works well only along the field lines, the low-\( \gamma \) case cannot allow for the thermal conduction effect throughout the system. Therefore, it may be misleading to say that the low-\( \gamma \) case corresponds to this phase of evolution. However, the low-\( \gamma \) model can still simulate some parts of the system in the presence of high-temperature region and large heat flow.) With these considerations in mind, we study the loop-top structure of less efficient thermal conduction by model

H1 and of efficient thermal conduction by model H2, respectively.

Figure 10a (Plate 26) is composed of the temperature contours \((\text{top left})\), the density contours \((\text{top right})\), the pressure contours \((\text{bottom left})\), and current density contours \((\text{bottom right})\) for model H1 at \( t = 10 \). Overlain are the magnetic field lines \((\text{solid lines})\) and the velocity fields \((\text{arrows})\). Figures 10b and 10c are the one-dimensional variations of representative physical quantities, such as fast-mode Mach number \((M_s)\), temperature \((T)\), current density \((J_y)\), gas pressure \((P_g)\), magnetic pressure \((P_m)\), density \((\rho)\), and the \( x \)-component of the magnetic field \((B_x)\), near the loop top along the \( z \)-axis and horizontal line, respectively.

The maximum temperature is reached near the neutral point, where the temperature is about 8 times as hot as that of the surroundings. There is little matter present near the neutral point, since plasmas in this region have been pushed away due to high magnetic tension force around the neutral point. The loop top is also hot; it is about 6 times as hot as the ambient region. Between these hot regions, there formed a triangular hot area \((-0.5, 1.7) \leq (x, z) \leq (0.5, 3.5)\) by slow and fast shocks (see Fig. 10a). Figure 10b shows clearly the formation of a fast shock and the density increases behind this, and Figure 10c indicates that there exists a hot and dense region between slow shocks. This high-temperature and high-density region is produced by the fast and slow shock and confined at the loop top.

Figures 11a–11c are the same as Figures 10a–10c but for model H2. The elapsed time is \( t = 19 \). Figure 11a (Plate 27) shows that temperature is relatively high both at the neutral point and in the loop top, but temperature enhancements are only higher by a factor of 2. On the other hand, the density enhancement at the loop top is larger than model H1; it is about 3 times denser than the ambient region (see Fig. 11c). In the low-\( \gamma \) case, therefore, temperature cannot increase much, while density can. Matter can thus be compressed more when \( \gamma \) is smaller.

Since this simulation does not include the radiative cooling effect, there remain hot regions inside the loop. But in reality, the cooling effect can easily decrease the temperature inside the loop to form Hz postflare loops (see § 6).

There is one noteworthy feature present in both Figure 10c and Figure 11c; the high-density region is confined by "magnetic pressure walls." Thus, the dense region can stay at the loop top for a long time, since the magnetic walls prevent the gas from draining downward.

3.5. Effect of \( \gamma \)

Finally we describe the \( \gamma \)-dependence. Figure 12 compares the time evolution of reconnection rates for the normal-\( \gamma \) case \((\gamma = 5/3)\) and the low-\( \gamma \) case \((\gamma = 1.1)\). Time and reconnection rate are normalized by \( t_{A0} \) and \( V_{A0} B_0 \), respectively. It is interesting to note that the evolution of the low-\( \gamma \) case is more violent than the normal-\( \gamma \) case; that is, the system undergoes a rapid increase and subsequent decrease in the reconnection rate. Therefore, if the thermal conduction works more efficiently, flares exhibits more drastic evolution. Figure 13 displays the time evolution of the ratios of the thermal and kinetic energies to the released magnetic energy for the normal-\( \gamma \) case and for the low-\( \gamma \) case, respectively. The formulation is

\[
\frac{\Delta E}{\Delta E_m} = \frac{|E(t) - E(0)|}{|E_m(t) - E_m(0)|},
\]

(20)
where $E(t)$ represents the kinetic or thermal energies integrated over a region of $(0, 0) \leq (r, z) \leq (8, 19.2)$, and $E_m$ is the magnetic energy ($t$ indicates time normalized by $t_{A0}$).

$$
E(t) = \begin{cases} 
\frac{1}{2} \int_0^{x_1} \int_0^{z_1} \rho \omega^2 \, dx \, dz & \text{for kinetic energy,} \\
\int_0^{x_1} \int_0^{z_1} \frac{\rho}{\gamma - 1} \, dx \, dz & \text{for thermal energy,}
\end{cases}
$$

$$
E_m(t) = \int_0^{x_1} \int_0^{z_1} \frac{B^2}{8\pi} \, dx \, dz,
$$

where $x_1$ and $z_1$ are 8.0 and 19.2, respectively. We determine the time span during which there is little outflow from the boundary. The earliest phase ($0 \leq t/t_{A0} \leq 2$) corresponds to the time range in which the initial perturbation proceeds. It is noticeable that 80% of the released magnetic energy is initially converted to the thermal energy and 20% to the kinetic energy in the normal-γ case (denoted by the thick line). While the thermal energy tends to decrease with time, the kinetic energy increases gradually, reaching about 40% of the released magnetic energy at maximum. This implies that a large amount of the released energy is lost in the form of kinetic energy. In the low-γ case (thin line), in contrast, the released magnetic energy is converted more to kinetic energy.

Next, we plot in Figures 14a (the normal-γ case) and 14b (the low-γ case) the decreasing rate of magnetic energy, the rate of work due to Lorentz force, the Joule heating rate, and the input magnetic flux crossing over the line at $x = 4$ with $0 \leq z \leq 19.2$, respectively. This time span is the same as Figure 13. Energies are integrated over the calculated region of $(0, 0) \leq (x, z) \leq (4, 19.2)$.

$$
\left[ \frac{\partial E_m}{\partial t} \right] = \left[ \frac{\partial}{\partial t} \int_0^{x_2} \int_0^{z_2} \frac{B^2}{8\pi} \, dx \, dz \right],
$$

the rate of work due to Lorentz force

$$
\int_0^{x_2} \int_0^{z_2} (J \times B) \cdot v \, dx \, dz,
$$

Joule heating rate

$$
\int_0^{x_2} \int_0^{z_2} \frac{\eta}{4\pi} \, J^2 \, dx \, dz,
$$

Input flux

where $x_2$ and $z_2$ are 4 and 19.2, respectively.

It is found that the $J \times B$ acceleration is dominant compared with the Joule heating, which is consistent with the
results of Sato & Hayashi (1979). Compared with the normal-$\gamma$ case, the low-$\gamma$ case shows the suppression of Joule heating and the enhancement of the rate of work due to the Lorentz force, which implies that the evolution of the low-$\gamma$ case is more dynamical. The oscillation patterns are due to the reflections of magnetohydrodynamic waves at the side boundaries ($x = \pm 8$).

4. COMPARISON WITH YOHKOH OBSERVATIONS

4.1. Loop-Top Sources

Masuda et al. (1994) found a hard X-ray impulsive source well above the soft X-ray flaring loop in the impulsive phase of flares. In the gradual phase, the more long-lived source continues to exist near the loop top. We try now to interpret these observational facts in relation to the present simulations.

Let us begin with the impulsive phase. It is very likely that a hard X-ray impulsive source may be formed near the loop top, where the reconnection jet emerging from the neutral point collides with the material inside the loop, thus forming a fast shock. From the simulation of the phase when the reconnection rate rapidly increases, in fact, we found that the loop-top has a high-temperature and high-density region (see Fig. 10) encompassed with the fast shock and slow shocks, which probably emit strong radiation.

It is plausible to assume that after being heated by the thermal conduction originating from near the neutral point, the chromospheric gas evaporates and rises along the loop, emitting soft X-rays. The soft X-ray flaring loop is thus formed. We suppose that the gradual phase corresponds to the phase in which slow and fast shocks are formed in the loop filled with evaporated plasmas. As for this phase, we compare the low-$\gamma$ case with observation because in this phase the thermal conduction becomes effective. Figure 11 shows that the loop top becomes about 3 times denser than the ambient plasma, and the “magnetic pressure walls” form around this high-density core (see Fig. 11). These walls can confine the high-density core at the loop top for a long time, explaining the presence of a long-lived loop-top source during the gradual phase (see Acton et al. 1992; Feldman et al. 1994). [In the present simulation, this core continued to exist to the end, say, $t \approx 60 t_{\rm AO}$, which corre-

\[ \Delta E / \Delta t = (j \times B) \cdot \nabla \]

\[ \text{Energy / time (normalized)} \]

\[ \text{time / } t_{\rm AO} \]

\[ \text{Joule heating} \]

\[ \text{Input flux} \]

\[ \text{Energy / time (normalized)} \]

\[ \text{time / } t_{\rm AO} \]

\[ \text{Joule heating} \]

\[ \text{Input flux} \]

\[ \text{Energy / time (normalized)} \]

\[ \text{time / } t_{\rm AO} \]

\[ \text{Joule heating} \]

\[ \text{Input flux} \]

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responds to several minutes if we assume $V_{A0} \sim 1000\ km\ s^{-1}$ and loop height $\sim 20,000\ km\ (L_0 \sim 6000\ km)$.

### 4.2. Motion of Plasmoid

Next we will discuss the motion of the plasmoid. The velocity of the plasmoid is observationally about $40-50\ km\ s^{-1}$ in the early phase (see Shibata 1995). The Alfvén velocity in the active region corona is estimated to be

$$V_{A0} \approx 3000(B/100\ G)\left(n_e/10^{10}\ cm^{-3}\right)^{-1/2} \ km\ s^{-1}.$$ 

Hence, $V_{\text{plasmoid}} \approx 0.1V_{A0}$. On the other hand, from our simulations, $V_{\text{plasmoid}}$ is about $0.2V_{A0}$ in the standard model ($\rho_e = 6\rho_0$). (This value is estimated in the early phase: $t_{A0} \sim 20$, see Fig. 7b.) The reasons for this discrepancy may be as follows: First, the initial density in the current sheet, $\rho_e$, may be much larger than the values we set. As mentioned above, we find that the velocity of the plasmoid can be reduced by an increase of $\rho_e$. In some cases the pressure dependence of the ejection velocity follows $V_{\text{plasmoid}}\propto \rho_e^{-1/2}$, the required density in the current sheet is $\rho_e \sim 40\ \rho_0$. In contrast, Figure 4 indicates that the reconnection processes (characterized by $V_{\text{low}}\propto \rho_e^{-1/2}$) rapidly affect by changing $\rho_e$. To conclude, we need to assign large $\rho_e$ values so that we can explain observed $V_{\text{plasmoid}}$ and reconnection rates simultaneously.

### 4.3. Inflow Velocity

Finally, we give one prediction concerning the inflow velocity to the neutral point. Table 3 shows the relation between the inflow velocity of the gas into the neutral point ($V_{\text{inflow}}$) and the rise velocity of the loop ($V_{\text{loop}}$) for different $\beta$ models. (We also plot mean velocities in Table 3.) From this table, we observe that the inflow velocity is nearly comparable with the rise velocity regardless of $\beta$. Since the inflow velocity has not yet been observed, this relation may be used as a prediction or a guide for the inflow velocity.

### 5. DISCUSSION

Now we are ready to discuss how the main features of solar flares can be understood physically in the framework of the reconnection model on the basis of our results in § 3.

We discuss first the time evolution of the system during the impulsive phase. It is found that the plasmas surrounding the neutral point as well as between the slow shocks are strongly heated. Hence, thermal conduction should transport energy efficiently from this region to the lower atmosphere along the field lines, which works as heating of the lower atmosphere. Nonthermal electrons accelerated by unknown mechanisms can also heat the lower atmosphere. Atmospheric gas will then be evaporated into the upper atmosphere, filling the magnetic loops, thereby making the soft X-ray flaring loops, although the present simulations performed for the basis of the fluid approximation are unable to describe such phenomenon directly. The temperature near the neutral point is $8$ times as large as the value outside the current sheet (see Fig. 10a). If the initial plasma $\beta$ is sufficiently low, $\beta \ll 1$, the temperature would be much higher because the converted thermal energy will increase. (Roughly, $T_{\text{neutral point}} \propto B_0^2/8\pi$, so $T_{\text{neutral point}}/T_{\text{outside}} \propto (B_0^2/8\pi)/T_{\text{outside}} \propto \beta^{-2}$.) Thus, it will be reasonable to think that such a very hot region can produce high-speed conduction flow.

Figure 10a shows that the loop-top region behind a fast shock has high density, about twice as dense as the ambient region. Temperature at the loop top is also enhanced by slow shocks and is about 6 times greater than that of the embedded region (Fig. 10c). The loop-top region is, hence, very likely to emit hard X-rays, which is consistent with the observations of the flares in the impulsive phase (see Masuda et al. 1994).

Next we discuss the gradual phase of the impulsive flares. Since thermal conduction and radiative cooling work well in this phase, the low-$\gamma$ case is relevant in this phase. From Figures 11a–11c, we find a high-density region in the loop top, where the magnetic fields sink at the center (on the line of $x = 0$) and encompass the matter. We suggest that this dense region could be the source with long-lived thermal emission observed in the gradual phase (see Acton et al. 1992; Feldman et al. 1994).

As for the $\gamma$ effect, the system with lower $\gamma$ shows a more drastic evolution (see Fig. 12) (see also related works by Sato et al. 1992 and Yokoyama & Shibata 1993). So, if we assume that the intensity of high-energy radiation, such as hard X-rays, relates to the reconnection rate, which is somewhat rough assumption, it follows that thermal conduction should be efficient in flares exhibiting a rapid intensity variations. This implies that one of the characteristics of the impulsive flare may be an effective thermal conductivity.

Through discussions above, we may conclude that the reconnection model for the cusp-type flare can also explain many features of non–cusp-type flares observed in soft and hard X-rays. This reconnection model has been proposed and developed mainly for the cusp-type flares (see, for example, Forbes & Priest 1982; Forbes 1985; Forbes & Malherbe 1985, 1988). Figure 15 summarizes our view of the cusp-type flare in the impulsive phase (left) and in the gradual phase or LDE flares (right), respectively (see also the discussion on the “unified model” by Shibata et al. 1995).

In the impulsive phase, there are downward flows of high-energy electrons and thermal conduction originating from the neutral point and directing along the magnetic fields towards the chromosphere. These downflows form the conduction front (see Forbes & Malherbe 1985) and heat the neighboring plasmas. The chromospheric gas is heated and evaporates into the corona. Since the emissivity is more sensitive to density than temperature, very hot regions will not always be observed, unless density is very high. Thus, we conjecture that the soft X-ray bright loop could be a dense region filled with evaporated gas. The reconnection jet collides with the loop top, forming the hot and dense core which will emit hard X-rays. If the evaporated flow has not yet reached this energetic region, the hard X-ray source above the soft X-ray flaring loop will be observed.

In the gradual phase or LDE flare, thermal conduction would be efficient, and the reconnected loop under the neutral point would be sufficiently filled with the evaporated flow. The conduction front will heat up the material along the boundary of the loop, which leads to its observation as a bright cusp-type structure in soft X-rays. The loop-top source is observed to be embedded by these evaporated gases. As for the inside loops, heating is no longer effective so that the loops can cool via radiative cooling process, thereby forming Hz post-flare loops.

The system in the impulsive phase has a small diffusion region and shows the effective “Petschek-type” configuration, while in the gradual phase it has a long elongated
diffusion region and shows the less effective "Petschek-type" configuration (see Petschek 1964). These configurations can explain the violent energy release in the impulsive phase and subsequent gradual relaxation in the gradual phase.

6. CONCLUSIONS

1. On the basis of the reconnection model, we have studied the structures of the cusp-type flares in detail. We found a high-temperature and density region at the loop top (below the neutral point), which is heated by slow shocks and confined by the magnetic pressure walls. These are in good agreement with the structures of the impulsive flares observed by Yohkoh.

2. A plasmoid with a high-density core is formed above the neutral point. It is surrounded by high-temperature boundary layers and moves upward. The motion of the plasmoid is quite sensitive to the initial density in the current sheet \( \rho_0 \). The observed plasmoid velocity can be reproduced when we assign \( \rho_0 \approx 40\rho_0 \). Alternatively, perpendicular magnetic fields, if they ever exist, would prevent the upward motion of the plasmoid.

3. We have investigated the effect of plasma \( \beta \) on the evolution of the flare and found that the results are almost independent of \( \beta \) if velocity and timescale are normalized by \( V_{A0} \) and \( t_{A0} \). This will help us to understand the nature of the flare in more realistic situations in which \( \beta \) is too small to study in the numerical simulation.

4. As reconnection proceeds, the height of the loop top moves upward with time as \( z \propto t^{0.7} \). Hence, the loop rises rapidly at first and is then gradually decelerated later. This is consistent with the observations (see Pneuman 1981).

5. The relation between the inflow velocity and the rise velocity of loop was studied. They have almost the same values regardless of \( \beta \).

6. When \( \gamma \) is near 1, which may simulate the case of efficient thermal conduction, the evolution of the system is more drastic than in the \( \gamma = 5/3 \) case. Moreover, plasma heating will be suppressed in general, but the compressional effect is more enhanced than in the \( \gamma = 5/3 \) case. In energetics, hence, magnetic energy is converted more efficiently into kinetic energy than thermal energy.

7. Our simulations can well account for the existence of the loop-top, hard X-ray sources discovered in the impulsive flares. We conclude that both the impulsive flares and the LDE flares can be understood in general by the reconnection model for the cusp-type flares.

More realistic calculations incorporating the effects of radiative cooling, thermal conduction, and perpendicular magnetic fields are necessary in future work, with which detailed comparisons with the observations will become possible. Moreover, the initial process of forming the current sheet should also be considered. This will help our understanding of the nature of solar flares in general.

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Fig. 6.—(a) Temperature contours of the plasmoid for the standard model (model S): solid lines and arrows represent magnetic field lines and velocity fields (normalized by $V_\infty$), respectively. (b) Same as (a), but for density contours. Elapsed time $t = 16.0(L_\infty/C_\infty)$ in both panels.

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FIG. 10.—(a) Temperature (top left), density (top right), pressure (bottom left), and current density (bottom right) contours of the loop (model H1). Solid lines and arrows represent field lines and the velocity fields (normalized by $V_{ke}$), respectively. (b) Variations of temperature ($T$), density ($\rho$), gas pressure ($P_g$), the absolute value of the $x$-component of the magnetic field ($|B_x|$), and fast-mode Alfvén Mach number ($M_a$) along the $z$-axis. (c) Variations of temperature ($T$), density ($\rho$), current density ($J_y$), gas pressure ($P_g$), and magnetic pressure ($P_m$) along the horizontal line crossing the loop top. Time $t = 100$ in all cases.

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Plate 27

Plate 11a

Same as Fig. 10, but for the low-γ model (model H2). Time t = 190 in all cases.

Plate 11b

Same as Fig. 10, but for the low-γ model (model H2). Time t = 190 in all cases.

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