The Scaling of Solar Flare Hard X-ray Emission to Other Flaring Objects in the Universe

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Abstract. Fletcher and Martens have successfully modeled solar hard X-ray sources observed at the top and footpoints of flaring magnetic loops with a Fokker-Planck type particle transport code. I show here that there are invariances in the Fokker-Planck equations that make these results applicable to environments with vastly different physical parameters, such as hard X-ray flares in accretion disks in active galactic nuclei, and in RS CVn and ALGOL type binaries.

The hard and soft X-ray telescopes onboard the Japanese Yohkoh spacecraft have unambiguously established that solar flare Hard X-ray (HXR) emission originates from the footpoints of magnetic loops, as well as from the tops of these same loops (Masuda et al., 1994).

Fletcher & Martens (1998) model the HXR emission from the looptops and the footpoints, using the commonly observed cusped loop geometry with a current-sheet on top, and they find a looptop and two footpoint HXR sources akin to those observed by Yohkoh. The Fletcher & Martens model relies on a numerical solution of the Fokker-Planck equation for the propagation of the electron beam injected at the looptop. In dimensionless units their model equation reads

\[
\frac{\partial f}{\partial t} + \mu v \frac{\partial f}{\partial z} - \varepsilon \frac{\partial}{\partial E} \left( f \frac{v}{E} \right) - \frac{v}{2} \frac{\partial}{\partial \mu} \left( (1 - \mu^2) \frac{d\ln B}{dz} f \right) \nonumber \\
- \varepsilon \frac{\partial}{\partial \mu} \left( (1 - \mu^2) \frac{\partial f}{\partial \mu} \right) = 0, \tag{1}
\]

where \( f(z,t,\mu) \) is the particle distribution function as function of time \((t)\), position along a fieldline \((z)\), and pitch-angle \((\mu)\), \( B \) is the magnetic field strength, \( E \) the particle energy, and \( v \) the particle velocity. This dimensionless form is found by defining the units (subscript zero) \( t_0 = L/v_0, \quad E_0 = \frac{m_e v_0^2}{2}, \quad \varepsilon = \frac{\pi e^2 A n_e L}{E_0} \).

Here \( L \) is the loop length from top to footpoint, \( A \) is the Coulomb-factor \((\approx 20)\), while \( m_e, n_e, \) and \( \varepsilon \) are the mass, density, and charge of the electrons.

The only parameter in this equation is \( \varepsilon \) which equals the ratio of the electron crossing time and the ionization time of the electron beam. Therefore \( \varepsilon \leq 1 \) implies the beam reaches the footpoints of a flaring loop only partially attenuated, and \( \varepsilon > 1 \) implies the beam will diffuse before reaching the footpoints. Typical values for the solar corona are \( L = 2 \times 10^9 \text{ cm}, \quad n_e = 4 \times 10^9 \text{ cm}^{-3} \), and \( E_0 = 30 \text{ keV}. \) Hence \( v_0 = c/3, \quad t_0 = 0.2 \text{ s}, \) and \( \varepsilon = 0.2. \)
The initial condition for the electron beam distribution function inferred from observations is a power law in energy, and a sharply peaked (beamed) function in pitch-angle, which is invariant for the choices of the dimensionality parameters that determine $\varepsilon$. Thus, with these initial conditions, the value of $\varepsilon$ completely determines the character of the solutions, and one can simply scale solutions from one part of parameter space to another for a given value of $\varepsilon$.

Let us examine two simple examples of scaling invariance. Consider two flaring loops, one five times longer than the other ($L_{\text{new}} = 5 \times L_{\text{old}}$), and keep the column depth, $n_e L$, and hence $\varepsilon$ constant. All solutions will be identical, except the new time unit, $t_0 = L_{\text{new}} / v_0$, is five times larger, i.e. the HXR evolution is five times slower.

In the second example I keep the electron density $n_e$ constant, and choose $L_{\text{new}} = 4 \times L_{\text{old}}$, and $E_{\text{0new}} = 2 \times E_{\text{0old}}$. Hence $\varepsilon$ is preserved, and with the same power-law initial conditions all solutions are identical in form, with the 30 keV emission profile shifted to 60 keV, and the time unit ($t_0 = L_{\text{new}} / v_0$) a factor $2\sqrt{2}$ times larger.

Typical parameters for magnetic loops in an accretion disk in an active galactic nucleus are $L \approx 3 \times 10^7$ cm, and $n_e \approx 2 \times 10^{18}$ cm$^{-3}$ (de Vries & Kuipers, 1992). Therefrom I find that $\varepsilon \approx 5 \times 10^5$, so that the flaring loop is completely optically thick for 30 keV electrons. One would observe a loop top source, but no footpoints, in the keV range. The energy spectrum is that of a thick target source, and typical timescales would be much shorter than in the solar case (since the loop length is much less). A loop top source originating from magnetic trapping, plus footpoint sources can only be expected for 60 MeV electrons (and GeV range protons), but the amount of particles generated at these energies is usually much lower than at keV ranges.

Typical members of the families of RS CVn and Algol type binaries are Algol itself, and $\sigma^2$ CrB. From it EXOSAT observations van den Oord, Mewe, and Brinkman (1988) find that Algol has magnetic loops with a density of the order of $3 \times 10^{11}$ cm$^{-3}$, and looplength $7.8 \times 10^{10}$ cm, while $\sigma^2$ CrB has respectively $3 \times 10^{11}$ cm$^{-3}$, and $4.5 \times 10^{10}$ cm. Their flare X-ray luminosities are respectively one and two orders of magnitude larger than that of the Sun. For both binary systems $\varepsilon \gg 1$ in the keV energy range, and hence once more one would observe a HXR loop top source, but no footpoints. HXR footpoints would emerge above about 1.5 MeV.

We conclude that the Fletcher & Martens (1998) simulations can be used without much adaptation to generate HXR energy spectra and evolution curves for a large variety of flaring astrophysical objects.

References

de Vries, M., Kuipers, J. 1992, 266, 77