## Sensitivity of MOSES

## to Chromospheric Spectral Line Profiles

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## Abstract

The Multi-Order Solar EUV Spectrograph (MOSES) is a new type of slitless imaging spectrograph with detectors at three spectral orders ( $n=-1,0,1$ ). The slitless configuration allows collection of spectroscopic information simultaneously over a 2D image. This study assesses the information content of an idealized MOSES data set by deriving and analyzing the nullspace of the instrument response. The conclusion is that MOSES is sensitive to line intensity, line width, and doppler shift but is insensitive to higher moments of the line profile. Addition of detectors at higher orders would impart sensitivity to line asymmetry and higher moments.

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## A 3-Order Slitless Spectrograph



Projection through $(x, \lambda)$ space


The projection from object $v(x, \lambda)$ to data $I=\left[I_{-}, I_{0}, I_{+}\right]$is a linear transform $U$,

$$
U v=I .
$$

Given an observation $I$, we would like to invert $U$ to obtain $v$. However, the data space is smaller than the object space. This leads to non-uniqueness: $I=U v=U v^{\prime}$.

Let $f=v^{\prime}-v$.

$$
\begin{equation*}
\therefore \quad U f=0 . \tag{1}
\end{equation*}
$$

The nullspace is the set of objects $f$ satisfying eq. 1. The better we understand the nullspace, the better we will understand the ambiguity introduced by non-uniqueness.

## The Generating Function

For a nullspace element $f$, we define a "generating function"

$$
\begin{equation*}
g(x, \lambda) \equiv \iiint f(x, \lambda) d \lambda d(\lambda+x) d(\lambda-x) \tag{2}
\end{equation*}
$$

Conversely, any function $g$ that has zero first and second partial derivatives at the upper and lower boudaries will give rise to a null function $f$ :

$$
\begin{equation*}
f(x, \lambda)=\left(\frac{\partial}{\partial \lambda}\right)\left(\frac{\partial}{\partial(\lambda+x)}\right)\left(\frac{\partial}{\partial(\lambda-x)}\right) g(x, \lambda) \tag{3}
\end{equation*}
$$

## Discrete Representation

Differential operators in continuous space may be represented as convolution kernels in discrete space:

$$
\begin{align*}
& \frac{\partial}{\partial \lambda} \longleftrightarrow\binom{1}{-1}, \\
& \frac{\partial}{\partial(\lambda+x)} \longleftrightarrow\left(\begin{array}{rr}
0 & 1 \\
-1 & 0
\end{array}\right), \\
& \frac{\partial}{\partial(\lambda-x)} \longleftrightarrow\left(\begin{array}{rr}
1 & 0 \\
0 & -1
\end{array}\right) .  \tag{4}\\
& \therefore\left(\frac{\partial}{\partial \lambda}\right)\left(\frac{\partial}{\partial(\lambda+x)}\right)\left(\frac{\partial}{\partial(\lambda-x)}\right) \leftrightarrow \kappa \equiv\left(\begin{array}{rrr}
0 & 1 & 0 \\
-1 & -1 & -1 \\
1 & 1 & 1 \\
0 & -1 & 0
\end{array}\right) . \tag{5}
\end{align*}
$$

## So What?

The nullspace consists of superpositions of $\kappa$ :

$$
\begin{equation*}
f=\kappa \otimes g . \tag{6}
\end{equation*}
$$

The $n$th moment of $\kappa$ is given by

$$
\sum_{x, \lambda}\left(\lambda-\lambda_{0}\right)^{n} \kappa_{x, \lambda} .
$$

| $n$ | Moment | Physical Interpretation |
| :---: | :---: | :---: |
| 0 | 0 | line intensity |
| 1 | 0 | doppler shift |
| 2 | 0 | line width |
| 3 | 6 | asymmetry |

Since the $n=0,1,2$ moments of $\kappa$ are zero, no combination of nullspace elements can alter the line intensity, doppler shift, or line width. Imaging at three orders therefore constrains exactly these three pieces of information.

## Demonstration



A test pattern was projected onto three slitless spectrograms and then reconstructed by Fourier backprojection with two added constraints: (1) the slit averaged spectrum and (2) non-negativity. As expected, the doppler shift and line width are preserved but the selfreversal is suppressed. A more sopthisticated reconstruction method is demonstrated in the poster by Fox \& Kankelborg.

## Conclusions

- A 3-order slitless spectrograph such as MOSES is sensitive to 3 line profile parameters: intensity, doppler shift and width.
- If we represent the solution in terms of line profile parameters and assume the skew and higher moments are zero, then much of the ambiguity due to non-uniqueness of the inverse problem will be eliminated.

The first sounding rocket flight of MOSES is planned for Spring, 2004. The instrument will image the chromosphere and corona in He II $\lambda 303.8$ Ảand Si XI $\lambda 303.3$ Ả, obtaining maps of doppler velocity and linewidth at $\sim 10$ s cadence.

