

MOSES Deconvolution

Charles C. Kankelborg
Department of Physics
Montana State University
Bozeman, MT 59717
kankel@mithra.physics.montana.edu

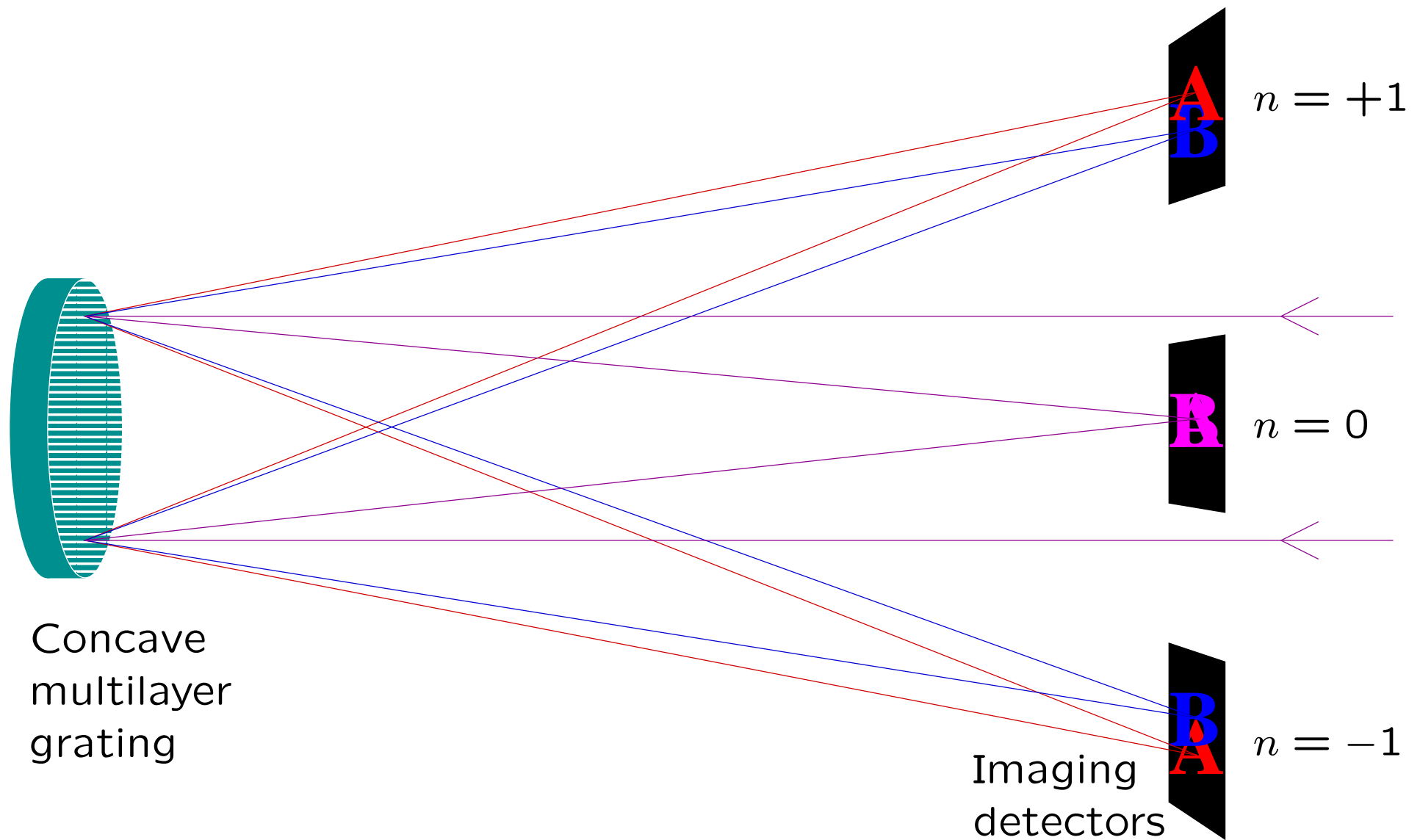
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Abstract

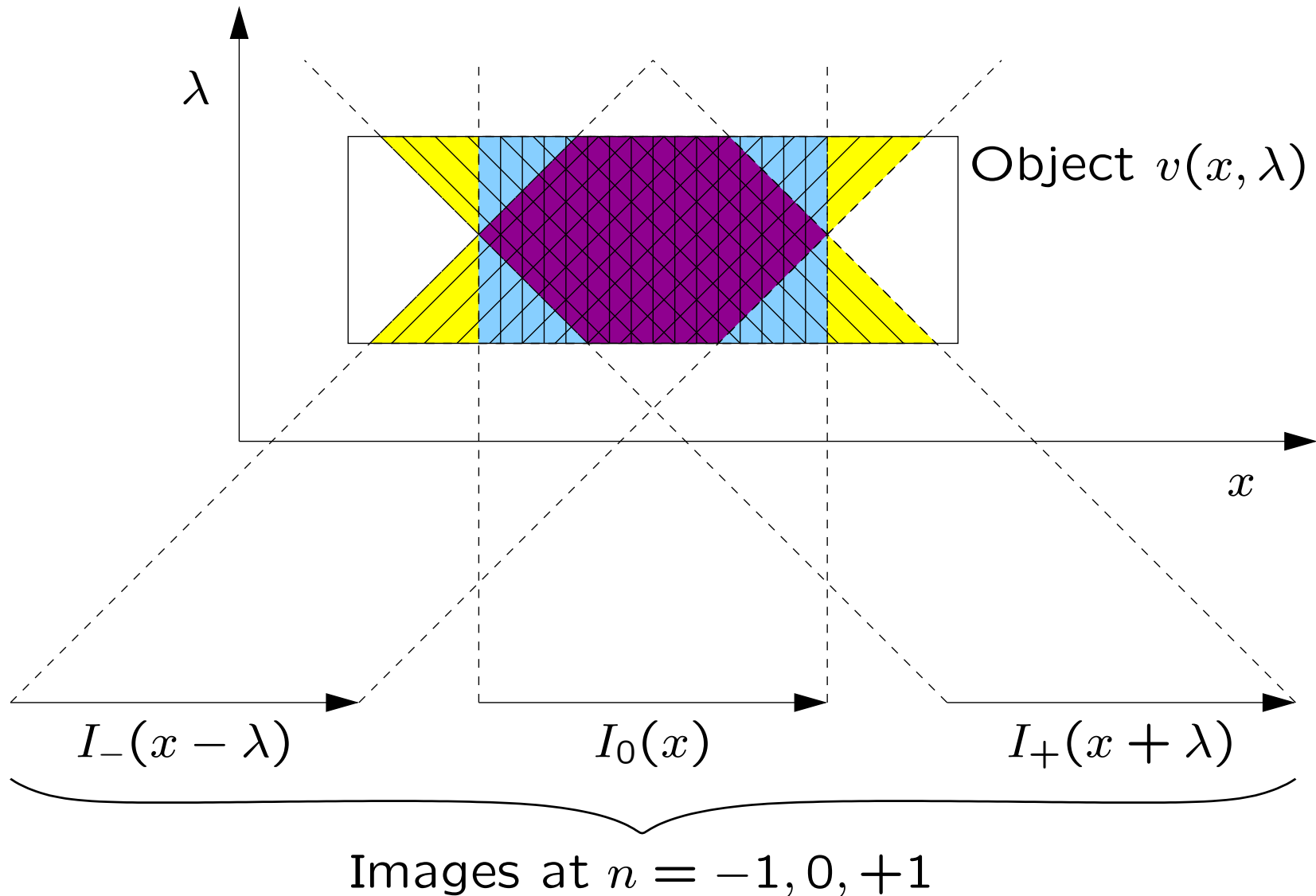
The *Multi-Order Solar EUV Spectrograph (MOSES)* is a rocket borne slitless imaging spectrograph with detectors at three spectral orders ($n = -1, 0, 1$). The algorithms to decode the *MOSES* data—to recover spectral information from the images—are still being developed. This presentation begins by describing mathematically the information content of the *MOSES* images. Several methods of image reconstruction are discussed. I'll also touch on related work by other investigators.

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A Multi-Order Slitless Spectrometer



Projection Through (x, λ) Space



The Big Question: Inversion

Can we turn $I_+(x + \lambda)$, $I_0(x)$, $I_-(x - \lambda)$ back into $I(x, \lambda)$? And if so, how?

The inversion problem seems poorly constrained: solve for $M \times N$ parameters from $M \times 3$ data points!

The projection from object $v(x, \lambda)$ to data $I = [I_-, I_0, I_+]$ is a linear transform U ,

$$Uv = I.$$

Given an observation I , we would like to invert U to obtain v . However, the data space is smaller than the object space. This leads to non-uniqueness: $I = Uv = Uv'$.

Let $f = v' - v$.

$$\therefore Uf = 0. \tag{1}$$

The **nullspace** is the set of objects f satisfying eq. 1. The better we understand the nullspace, the better we will understand the ambiguity introduced by non-uniqueness.

The Generating Function

For a nullspace element f , we define a “generating function”

$$g(x, \lambda) \equiv \iiint f(x, \lambda) d\lambda d(\lambda + x) d(\lambda - x). \quad (2)$$

Conversely, any function g that has zero first and second partial derivatives at the upper and lower boundaries will give rise to a null function f :

$$f(x, \lambda) = \left(\frac{\partial}{\partial \lambda} \right) \left(\frac{\partial}{\partial(\lambda + x)} \right) \left(\frac{\partial}{\partial(\lambda - x)} \right) g(x, \lambda). \quad (3)$$

Discrete Representation

Differential operators in continuous space may be represented as convolution kernels in discrete space:

$$\begin{aligned}
 \frac{\partial}{\partial \lambda} &\longleftrightarrow \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \\
 \frac{\partial}{\partial(\lambda + x)} &\longleftrightarrow \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \\
 \frac{\partial}{\partial(\lambda - x)} &\longleftrightarrow \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.
 \end{aligned} \tag{4}$$

$$\therefore \left(\frac{\partial}{\partial \lambda} \right) \left(\frac{\partial}{\partial(\lambda + x)} \right) \left(\frac{\partial}{\partial(\lambda - x)} \right) \leftrightarrow \kappa \equiv \begin{pmatrix} 0 & 1 & 0 \\ -1 & -1 & -1 \\ 1 & 1 & 1 \\ 0 & -1 & 0 \end{pmatrix}. \tag{5}$$

So What?

The nullspace consists of superpositions of κ :

$$f = \kappa \otimes g. \quad (6)$$

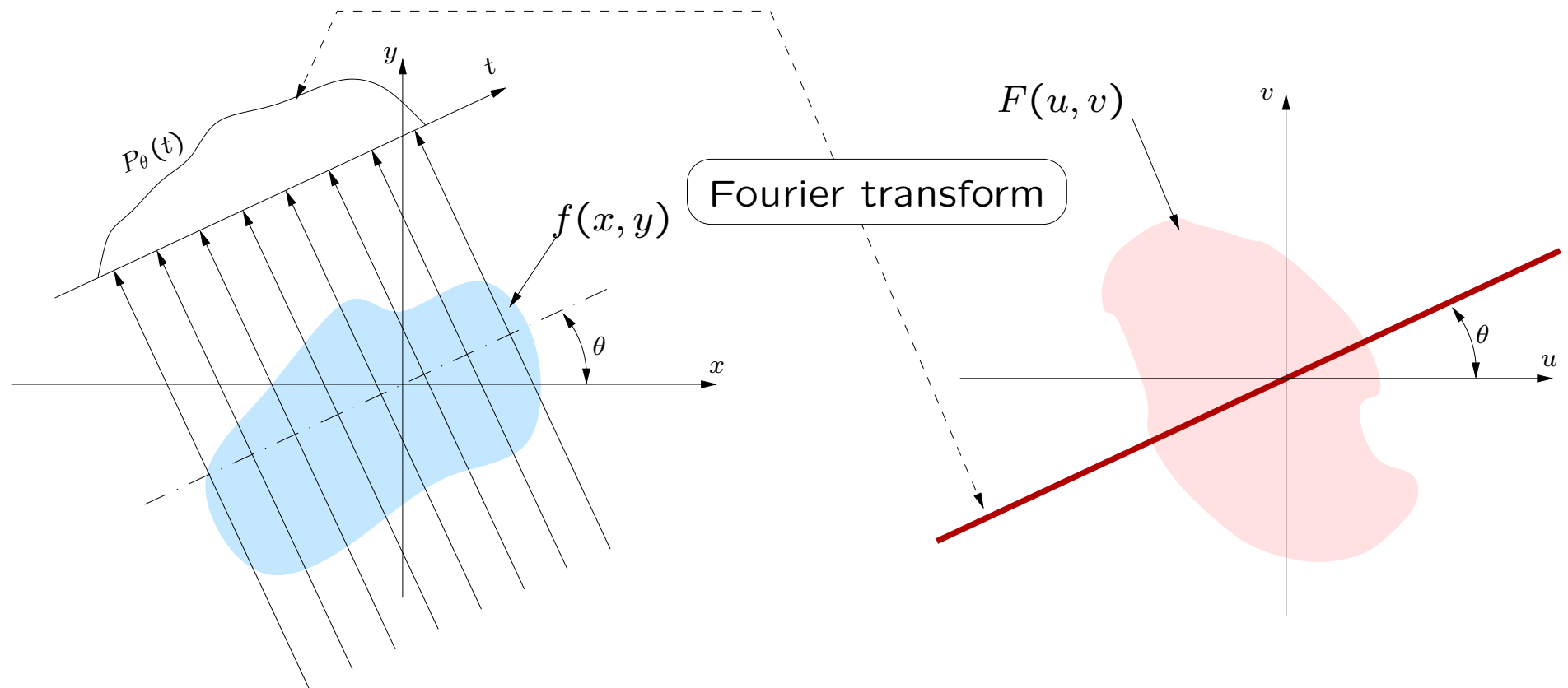
The n th moment of κ is given by

$$\sum_{x,\lambda} (\lambda - \lambda_0)^n \kappa_{x,\lambda}.$$

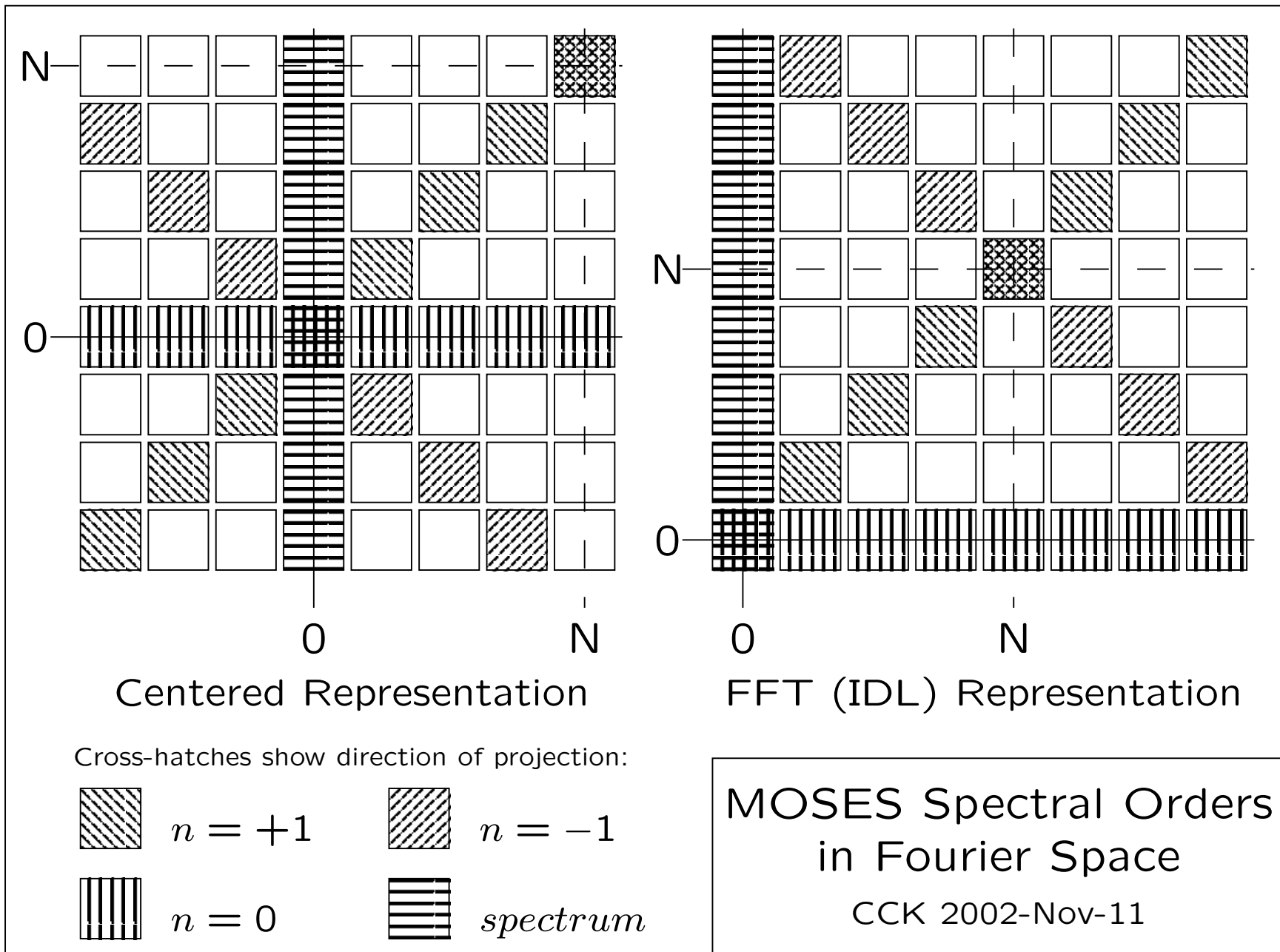
n	Moment	Physical Interpretation
0	0	line intensity
1	0	doppler shift
2	0	line width
3	6	asymmetry

Since the $n = 0, 1, 2$ moments of κ are zero, no combination of nullspace elements can alter the line intensity, doppler shift, or line width. Imaging at three orders therefore constrains exactly these three pieces of information. *Strictly speaking, this applies only to the average in x .*

The **Fourier Slice Theorem** is the fundamental theorem of tomography.*



*See A.C. Kak and Malcolm Slaney, *Principles of Computerized Tomographic Imaging*, IEEE Press, 1988. Available free: <http://www.slaney.org/pct/>



Reconstruction Algorithms

Algorithms of several types are being studied:

- Parametric modeling (Fox and Kankelborg)
- Singular value decomposition (Fox and Vogel)
- Pixon reconstruction (Fox and Metcalf)
- Fourier backprojection (Kankelborg)

A quick-and-dirty Fourier backprojection algorithm will be discussed in detail.

MOSES Backprojection Algorithm

1. Create a blank image $F(u, v)$ that will hold the FFT of the reconstructed image $I(x, \lambda)$.
2. Take the Fourier Transform $S_n(k)$ of each of the three projections I_+, I_-, I_0 .
3. Use the F_n to fill in the appropriate slices of $F(u, v)$.
4. Apply the inverse transform to obtain an initial guess $I(x, \lambda)$.

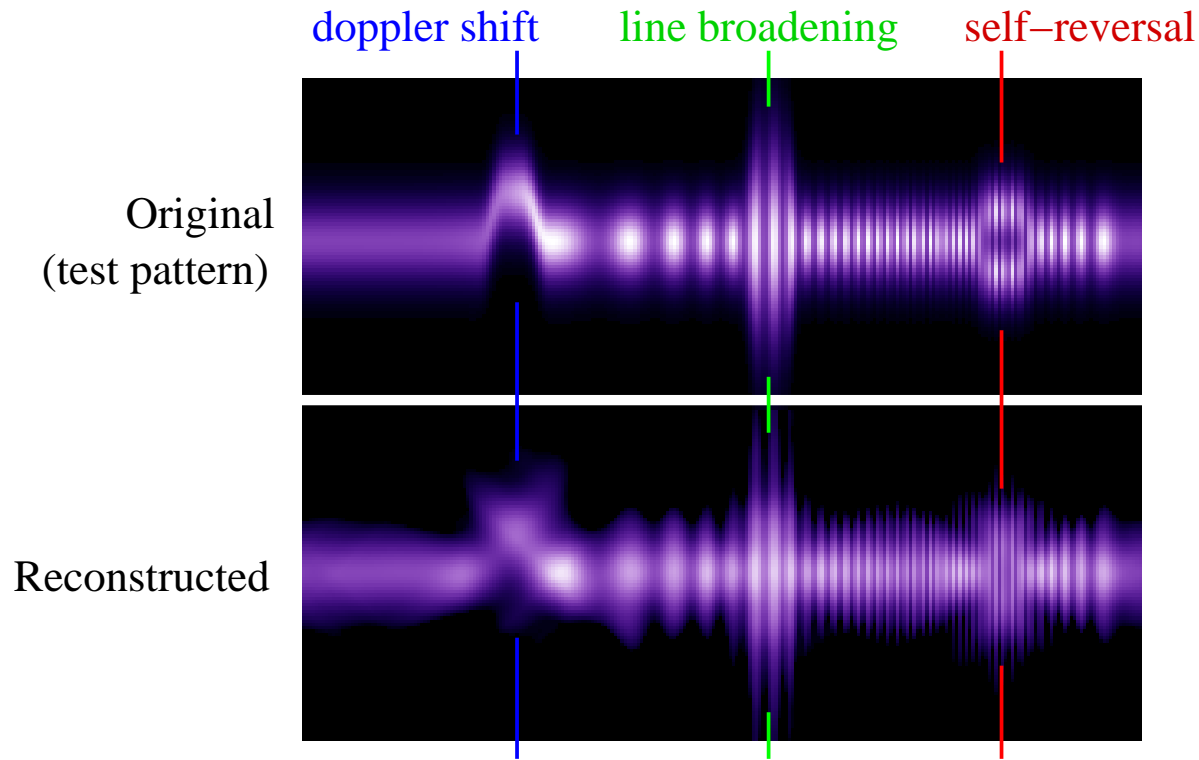
In practice, it helps to know (or be able to guess) the projection along x , $\int_{-\infty}^{\infty} I(x, \lambda) dx$.

Problem: lots of negative counts in the backprojected image.

We may use an iterative approach to enforce non-negativity:

1. Zero all negative elements of the backprojection.
2. Fourier transform the result, use it to fill the wedges of $F(u, v)$ that are not constrained by the data.
3. Inverse Fourier transform to obtain an improved backprojection.
4. Repeat until the sum of the negative counts in the backprojection is small compared to the total uncertainty in image counts.
5. Zero all negative pixels in the backprojection.

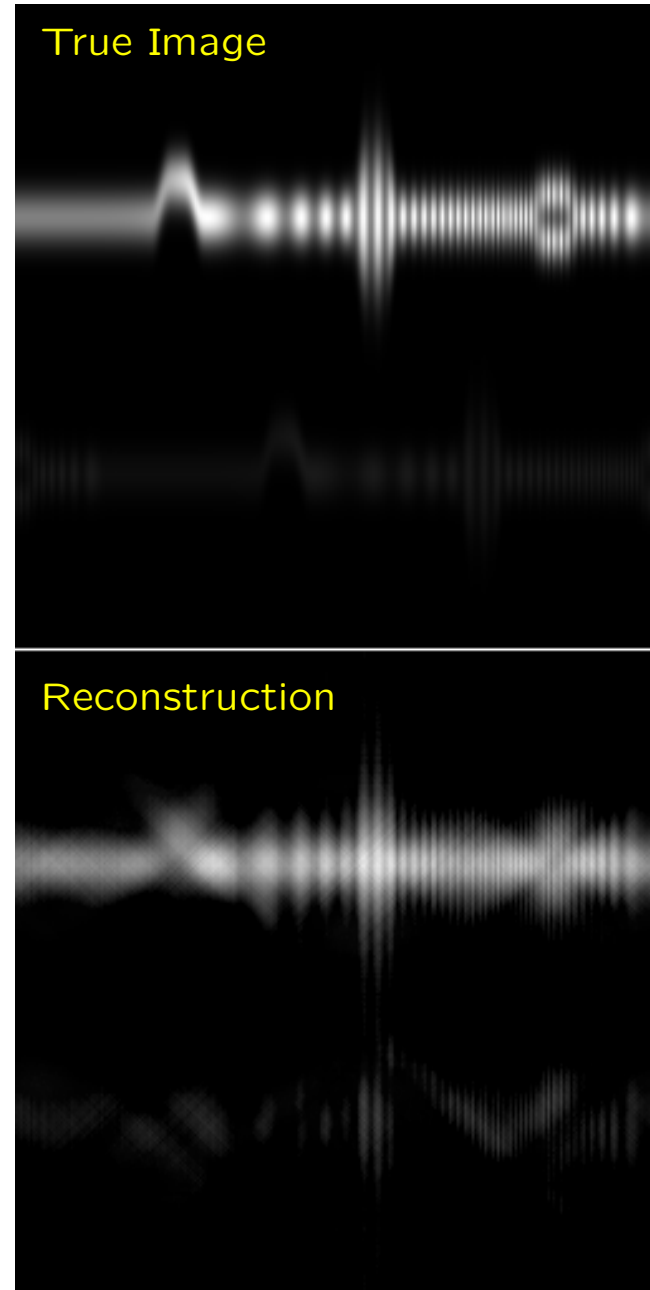
Data Inversion Example



A test pattern is projected onto the three spectrograph orders and then reconstructed by Fourier backprojection with two added constraints: (1) the slit averaged spectrum and (2) non-negativity. The doppler shift and line width are preserved but the self-reversal is suppressed.

Limitations of Fourier Backprojection

An example with two spectral lines. This time, photon noise is included. The bright line is reconstructed well, but the faint line is gibberish. We would do better with an algorithm that uses fewer degrees of freedom in the representation of faint objects.



Related Work

Several investigators are pursuing similar exciting ideas:

- Descour, M.R., *et al.* 1997: “Demonstration of a Computed-Tomography Imaging Spectrometer Using a Computer-generated Hologram Dispenser”, *Appl. Opt.* **36**: 16, 3694.
- Wilson, D.W., Maker, P.D., & Muller, R.E. 1997: “Reconstructions of Computed-Tomography Imaging Spectrometer Image Cubes”, *Proc. SPIE* **3118**, 184.
- DeForest, C.E. 2002: “Stereoscopic Spectroscopy: A New Technique for Rapid Magnetography”, *Proc. ASP, Solar and Stellar Magnetism III, in press*]

Conclusions

- A 3-order slitless spectrograph such as *MOSES* is sensitive to 3 line profile parameters: intensity, doppler shift and width.
- Much of the ambiguity due to non-uniqueness of the inverse problem will be eliminated by thinking in terms of moments of the line profile.
- Preliminary results with a simple Fourier backprojection algorithm look promising.
- *MOSES* is part of a broader trend toward computed tomographic imaging spectroscopy.