MOSES: The Instrument and The Math

Charles Kankelborg
Physics Department
Montana State University

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A pioneering remote sensing experiment is being developed at MSU: the *Multi-Order Solar Extreme Ultraviolet Spectrograph (MOSES)*. The *MOSES* rocket payload is scheduled for launch from White Sands Missile Range in August of this year. But the success of the mission hinges on what happens after the payload parachutes safely back to Earth: the mathematical and computational challenge of analyzing a new kind of data.
MOSES: Multi-Order Solar EUV Spectrograph

- NASA sounding rocket payload.
- Built at MSU, largely by students.
- Launch August 19, 2004 from WSMR.
- Extreme Ultraviolet (EUV) observations of the Sun
- Novel “snapshot” imaging spectrometer.
- Data interpretation: inverse problem
Suborbital Rocket (NASA’s part)

- Terrier booster, Black Brant VC sustainer
- Launch from White Sands Missle Range
- Apogee near 300 km
- 160 km upleg at 100 s; 300 s to obtain data
- Impact 80 km downrange
The Payload

- Guidance, telemetry, solar pointing and recovery (NASA)
- Experiment section (our part!)
MOSES instrument layout

- Read-Out Electronics
- Concave Grating
- Flat Mirror
- CCD Housing (1 of 3)
- Optical Table
MOSES flight hardware

Read-Out Electronics
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Read-Out Electronics

Concave Grating

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EUV Solar Transition Region

The Sun in He II, $\sim 80,000^{\circ}K$
Example: Flare Observation

2001/04/03 01:19
Imager or...

Pretty picture, not very quantitative.
...Spectrometer

Velocities and temperatures (but only along a slit).
"Rastering" multiple exposures to build an image.
…Spectrometer

“Rastering” multiple exposures to build an image.
Alternative: *MOSES*

Spectra over entire field in a single exposure!
Multi-order concept

Concave multilayer grating

n = +1

n = 0

n = -1

Imaging detectors
Inversion = Tomography

\[ I_n(x) = I_0(x) + (x - \lambda) \]

Images at \( n = -1, 0, +1 \)
Disentangling spatial and spectral information

Can we turn the four projections $I_+(x+\lambda), I_0(x), I_-(x-\lambda)$ and $I_\infty(\lambda)$ back into $I(x, \lambda)$? And if so, how?

The inversion problem seems poorly constrained: solve for $M \times N$ parameters from $M \times 4$ pieces of information!
Reconstruction Algorithms

- Parametric modeling
- Singular value decomposition
- Pixon reconstruction
- Fourier backprojection
- Algebraic reconstruction

Fourier backprojection and Algebraic reconstruction will be discussed in detail.
Fourier Slice Theorem

\[ F(u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) e^{-2\pi i (ux + vy)} \, dx \, dy \]

\[ f(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u, v) e^{2\pi i (ux + vy)} \, du \, dv \]

\[ P_\theta(t) \]

\[ \theta \]

\[ F(u, v) \]

\[ u \]

\[ v \]

\[ t \]

\[ x \]

\[ y \]
Fourier Backprojection

1.
1. Create a blank image $F(u, v)$ that will hold the FFT of the reconstructed image $I(x, \lambda)$. 
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2. Take the Fourier Transform $F_n(k)$ of each of the four projections $I_+, I_-, I_0, I_\infty$. 

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3. Use the $F_n$ to fill in the appropriate slices of $F(u, v)$. 
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2. Take the Fourier Transform $F_n(k)$ of each of the four projections $I_+, I_-, I_0, I_\infty$.

3. Use the $F_n$ to fill in the appropriate slices of $F(u, v)$.

4. Apply the inverse transform to obtain an initial guess $I(x, \lambda)$. 

Iterating for non-negativity

**Problem:** negative counts in the backprojected image.

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2. Fourier transform the result, use it to fill the wedges of $F(u, v)$ that are not constrained by the data.
Iterating for non-negativity

**Problem:** negative counts in the backprojected image.

1. Zero all negative elements of the backprojection.
2. Fourier transform the result, use it to fill the wedges of $F(u, v)$ that are not constrained by the data.
3. Inverse Fourier transform to obtain an improved backprojection.
Iterating for non-negativity

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4. Repeat until the sum of the negative counts in the backprojection is small compared to the total uncertainty in image counts.
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4. Repeat until the sum of the negative counts in the backprojection is small compared to the total uncertainty in image counts.
5. Zero all negative pixels in the backprojection.
A test pattern is projected onto the three spectrograph orders and then reconstructed by Fourier backprojection with two added constraints: (1) the slit averaged spectrum and (2) non-negativity. The doppler shift and line width are preserved but the self-reversal is suppressed.
Smooth Multiplicative Algebraic Reconstruction Technique

1.
1. $G(x, \lambda) = I_0(x) I_\infty(\lambda)/N$  
Initial guess.
Smooth Multiplicative Algebraic Reconstruction Technique

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2. $[I'_0, I'_+, I'_-, I'_\infty] = T G$ Projection.
1. \( G(x, \lambda) = I_0(x) I_\infty(\lambda)/N \)  
   Initial guess.

2. \([I'_0, I'_+, I'_-, I'_\infty] = TG\)  
   Projection.

3. \( G = G \left[ \frac{I_0(x)}{I'_0(x)} \right]^\gamma \cdot \left[ \frac{I_+(x+\lambda)}{I'_+(x+\lambda)} \right]^\gamma \cdot \left[ \frac{I_-(x-\lambda)}{I'_-(x-\lambda)} \right]^\gamma \cdot \left[ \frac{I_\infty(\lambda)}{I'_\infty(\lambda)} \right]^\gamma \)  
   Correction.
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4. \( G = G \otimes \frac{1}{1+2a+2b} \begin{bmatrix} 0 & a & 0 \\ b & 1 & b \\ 0 & a & 0 \end{bmatrix} \) \quad \text{Smoothing.}
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3. \( G = G \left[ I_0(x) \over I'_0(x) \right]^{\gamma} \cdot \left[ I_+(x+\lambda) \over I'_+(x+\lambda) \right]^{\gamma} \cdot \left[ I_-(x-\lambda) \over I'_-(x-\lambda) \right]^{\gamma} \cdot \left[ I_\infty(\lambda) \over I'_\infty(\lambda) \right]^{\gamma} \) Correction.

4. \( G = G \otimes \frac{1}{1+2a+2b} \begin{bmatrix} 0 & a & 0 \\ b & 1 & b \\ 0 & a & 0 \end{bmatrix} \) Smoothing.

5. \( [I'_0, I'_+, I'_-, I'_\infty] = T G \) Projection.

6. Evaluate goodness of fit, \( \chi_0^2, \chi_+^2, \chi_-^2 \).
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6. Evaluate goodness of fit, \( \chi_0^2, \chi_+^2, \chi_-^2 \).

7. Adjust smoothing parameters, \( a = a/(\chi_+^2 \chi_-^2); b = b/\chi_0^2 \).
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4. $G = G \otimes \frac{1}{1+2a+2b} \begin{bmatrix} 0 & a & 0 \\ b & 1 & b \\ 0 & a & 0 \end{bmatrix}$ Smoothing.
5. $[I'_0, I'_+, I'_-, I'_\infty] = T G$ Projection.
6. Evaluate goodness of fit, $\chi^2_0$, $\chi^2_+$, $\chi^2_-$. 
7. Adjust smoothing parameters, $a = a/(\chi^2_+ \chi^2_-)$; $b = b/\chi^2_0$.
8. Loop to (3).
Comparison of Algorithms

Differences between true and reconstructed line profile parameters. Line center and linewidth errors are in pixels. For each parameter, a mean offset and RMS error are given. One MOSES pixel is approximately 20 mÅ (20 km/s at 304 Å).

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>center (median) mean</th>
<th>RMS</th>
<th>width (quartiles 1-3) mean</th>
<th>RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fourier</td>
<td>-0.01</td>
<td>0.32</td>
<td>-0.19</td>
<td>0.97</td>
</tr>
<tr>
<td>SMART</td>
<td>0.04</td>
<td>0.23</td>
<td>0.18</td>
<td>0.34</td>
</tr>
</tbody>
</table>
Doppler Shift Estimation

He II median line center (pixels)

Reconstructed

SERTS-99

9.8
10.0
10.2
10.4
10.6
10.8
11.0

9.5 10.0 10.5 11.0 11.5
The *MOSES* rocket project is sponsored by NASA grant NAG5-10997.