

Explaining the Latitudinal Distribution of Sunspots with Deep Meridional Flow (Materials and methods)

Dibyendu Nandy and Arnab Rai Choudhuri

1 The basic equations

Calculations reported here were done with a code for solving the axisymmetric kinematic dynamo problem. An axisymmetric magnetic field can be represented in the form

$$\mathbf{B} = B(r, \theta)\mathbf{e}_\phi + \nabla \times [A(r, \theta)\mathbf{e}_\phi], \quad (1)$$

where $B(r, \theta)$ and $A(r, \theta)$ respectively correspond to the toroidal and poloidal components. The standard equations for the so-called $\alpha\Omega$ dynamo problem are:

$$\frac{\partial A}{\partial t} + \frac{1}{s}(\mathbf{v} \cdot \nabla)(sA) = \eta \left(\nabla^2 - \frac{1}{s^2} \right) A + \alpha B, \quad (2)$$

$$\frac{\partial B}{\partial t} + \frac{1}{r} \left[\frac{\partial}{\partial r}(rv_r B) + \frac{\partial}{\partial \theta}(v_\theta B) \right] = \eta \left(\nabla^2 - \frac{1}{s^2} \right) B + s(\mathbf{B}_p \cdot \nabla)\Omega + \frac{1}{r} \frac{\partial \eta}{\partial r} \frac{\partial}{\partial r}(rB), \quad (3)$$

where $s = r \sin \theta$. Here \mathbf{v} is the meridional flow, Ω is the internal angular velocity of the Sun, η is the turbulent diffusion and α is the coefficient which describes the generation of poloidal field at the solar surface from the decay of bipolar sunspots. We describe below how we specify \mathbf{v} , Ω , η and α . Once these quantities are specified, we can solve (2) and (3) to study the behavior of the dynamo. Apart from the specification of these parameters, we also include magnetic buoyancy in a way described below. We carry out our calculations in a quadrant $0.55R_\odot < r < R_\odot$, $0 < \theta < \pi/2$.

The boundary conditions are as follows. At the pole ($\theta = 0$)

$$A = 0, \quad B = 0, \quad (4)$$

whereas at the equator ($\theta = \pi/2$)

$$\frac{\partial A}{\partial \theta} = 0, \quad B = 0. \quad (5)$$

For a perfectly conducting solar core, the bottom boundary ($r = R_b$) condition should be

$$A = 0, B = 0. \quad (6)$$

At the top ($r = R_\odot$), the toroidal field has to be zero ($B = 0$) and A has to match smoothly to a potential field satisfying the free space equation

$$\left(\nabla^2 - \frac{1}{r^2 \sin^2 \theta} \right) A = 0. \quad (7)$$

We have used a finer grid resolution as compared to other models existing in the literature, with 128×128 grid cells in the latitudinal and radial directions. If the α coefficient has a quenching factor, then any arbitrary initial condition either asymptotically decays (sub-critical condition) or relaxes to a steady dynamo solution (super-critical condition).

2 Specification of parameters and magnetic buoyancy

The depth-dependent diffusivity in our model is given by

$$\eta(r) = \eta_{RZ} + \frac{\eta_T}{2} \left[1 + \operatorname{erf} \left(\frac{r - r_t}{d_t} \right) \right]. \quad (8)$$

The diffusivity η_T within the SCZ proper, which is predominantly due to turbulence, is taken to have the value $\eta_T = 0.8 \times 10^8 \text{ m}^2 \text{ s}^{-1}$. The diffusivity below the SCZ is much smaller and we take $\eta_{RZ} = 2.2 \times 10^5 \text{ m}^2 \text{ s}^{-1}$. The other parameters are $d_t = 0.05 R_\odot$ and $r_t = 0.7 R_\odot$.

The meridional circulation is obtained from the stream function ψ defined through the equation

$$\rho \mathbf{v} = \nabla \times [\psi(r, \theta) \mathbf{e}_\phi]. \quad (9)$$

We take

$$\psi r \sin \theta = \psi_0 (r - R_p) \sin \left[\frac{\pi(r - R_p)}{(R_\odot - R_p)} \right] \{1 - e^{-\beta_1 r \theta^\epsilon}\} \times \{1 - e^{\beta_2 r(\theta - \pi/2)}\} e^{-((r - r_0)/\Gamma)^2} \quad (10)$$

with the following values of the parameters: $\beta_1 = 1.65 \times 10^{-8} \text{ m}^{-1}$, $\beta_2 = 2.2 \times 10^{-8} \text{ m}^{-1}$, $\epsilon = 2.0000001$, $r_0 = (R_\odot - R_b)/4.15$, $\Gamma = 3.47 \times 10^8 \text{ m}$, $\gamma = 0.95$. The meridional flow not going below the tachocline was generated by taking $R_p = 0.675 R_\odot$, whereas we had taken $R_p = 0.6 R_\odot$ to produce a meridional flow going below the tachocline. The flow amplitude is specified in such a manner that a maximum flow speed of 20 m s^{-1} is reached at mid-latitudes near the surface.

We use the following analytic form to represent the solar internal rotation:

$$\Omega(r, \theta) = \Omega_{RZ} + \frac{1}{2} \left[1 + \operatorname{erf} \left(2 \frac{r - r_t}{d_t} \right) \right] [\Omega_{SCZ}(\theta) - \Omega_{RZ}]. \quad (11)$$

This analytical expression fits the results of helioseismology fairly closely for the following values of parameters: $r_t = 0.7R_\odot$, $d_t = 0.05R_\odot$, $\Omega_{RZ}/2\pi = 432.8$ nHz, $\Omega_{SCZ}(\theta) = \Omega_{EQ} + \alpha_2 \cos^2(\theta) + \alpha_4 \cos^4(\theta)$, with $\Omega_{EQ}/2\pi = 460.7$ nHz, $\alpha_2/2\pi = -62.69$ nHz and $\alpha_4/2\pi = -67.13$ nHz.

The α coefficient is taken in the form

$$\alpha = \frac{\alpha_0}{1 + (B/B_0)^2} \cos \theta \cdot \frac{1}{4} \left[1 + \operatorname{erf} \left(\frac{r - r_1}{d_1} \right) \right] \times \left[1 - \operatorname{erf} \left(\frac{r - r_2}{d_2} \right) \right]. \quad (12)$$

The parameters are $r_1 = 0.95R_\odot$, $r_2 = R_\odot$, $d_1 = d_2 = 0.025R_\odot$, making sure that the α effect is concentrated in the top layer $0.95R_\odot \leq r \leq R_\odot$. The α -quenching factor $1 + (B/B_0)^2$ included in the denominator helps the system to relax to periodic solutions with amplitude $B \sim B_0 = 10^5$ G. A value of about 1 m s^{-1} for α_0 keeps the solutions super-critical.

Finally, the buoyancy algorithm is implemented in the following fashion. We search for toroidal field B exceeding the critical field $B_c = 10^5$ G, *above (and not below)* the base of the SCZ ($0.71R_\odot$) at intervals of time $\tau = 8.8 \times 10^5$ s. Wherever B exceeds B_c , a fraction $f = 0.5$ of it is made to erupt to the surface layers—with the toroidal field values adjusted appropriately to ensure flux conservation.

Dynamo solutions for a deeply penetrating meridional flow (beneath the tachocline) are found to be fairly robust with respect to the choice of parameter values for η_T , α_0 and the meridional flow amplitude.