# Fast DEMs for EIS and AIA

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#### Introduction

Solar Coronal emission at a given temperature can be expressed in terms of a 'Differential Emission Measure' (DEM)  $I(T) \equiv n^2(T)$ , i.e., the squared density per unit temperature integrated along the line of sight. Observed intensities,  $C_n$ , in AIA or some other instrument are given by the convolution of an instrument response,  $R_n(T)$ , with the DEM being observed:

$$C_n = \int R_n(T)I(T)dT$$

We present a method for inferring DEMs using data from solar imagers such AIA. In its basic form, the method is very fast ( $\sim 1$  minute per full disk AIA image), although the DEMs obtained can contain regions of moderately negative emission measure (EM). We demonstrate an extension of the method which removes negative EM while still matching the data. The fidelity of the method is analyzed and it is applied to a coronal loop observed on April 19, 2011.

#### Basic Method

Assume that the DEM can be expressed as a linear combination of some set of basis elements,  $B_n$ , so that the integral equation becomes a matrix equation:

$$C_n = \int R_n(T)I(T)dT = \sum_k i_k \int R_n(T)B_k(T)dT \equiv \sum_k i_k \gamma_n A_{nk}, \quad (2)$$

# Example AIA Test DEM Inversions

(1)



# Example EIS Test DEM Inversions



Figure: Test DEM inversions using Log-normal simulated DEMs, convolved with the AIA response functions, with noise (Solid Lines), each with a width of 0.1.

#### Broader Distribution, Moderate EM



Figure: Test DEM inversions using Log-normal simulated DEMs, convolved with the EIS response functions, with noise (Solid Lines), each with a width of 0.1. 5 Spectral lines were used: Fe IX 188.497, Fe X 184.537, Fe XII 195.119, Fe XV 284.163, and Fe XVI 262.976.

EIS Response: Broader Distribution, Moderate EM

Chi squared percentiles; EM=5.0e+28, Width=0.15	Average Time per Pixel
	0.003

Where  $\gamma_n$  are a set of normalization constants for the response functions, which we choose to be their squared integral, square root. If the number of basis elements is equal to the number of instrument channels, it is straightforward to invert the equation to find the DEM coefficients,  $i_k$ . In that case, the quality of the inversion is highly sensitive to choice of basis.

An alternative is a large number of narrow temperature bins. The underconstrained inversion is resolved by a Singular Value Decomposition (SVD), which picks the coefficient vector with the smallest magnitude (i.e., least squared emission measure).

Remarkably, use of the (square-normalized) instrument response functions as a basis gives identical results. This reduces the number of operations required for each pixel to only  $N^2$  operations with N instrument channels. It also renders  $A_{ij}$  symmetric.

#### Regularization

The basic method's disadvantage is that it can produce negative coefficients and therefore negative EM. Therefore, we enforce a  $\chi^2$  threshold and seek a new DEM which minimizes the sum

$$\chi^2 + \lambda \int [I(T)]^2 dT = \sum_i \frac{\Delta C_i}{\sigma_i^2} + \lambda \sum_{ij} i_i i_j A_{ij}, \qquad (3)$$

where  $\chi^2$  has the usual definition, I(T) is the DEM described above, and  $\lambda$  is a regularization parameter chosen to enforce the desired  $\chi^2$  threshold,  $\chi^2_0$ .

Using our basic DEM solution, we replace  $i_i$  with a set of corresponding regularization corrections to the data values,  $\Delta C_i$ , so that  $i_i = \sum_i A_{ii}^{-1} (C_i + \Delta C_i) / \gamma_i$ :

$$\chi^2 + \lambda \int [I(T)]^2 dT = \sum_i \frac{\Delta C_i^2}{\sigma_i^2} + \lambda \sum_{ii} (C_i + \Delta C_i) \frac{A_{ij}^{-1}}{\gamma_i \gamma_j} (C_j + \Delta C_j).$$
(4)

Figure: Response to Log-normal distributions of width 0.15 and total EM  $5.0 \times 10^{28}$  at temperatures from 5.5 to 7.0 dex. Solid line on left shows emission measure weighted median temperature (EMWMT).

# Narrow Distribution, Lower EM



Figure: Response to Log-normal distributions of width 0.1 and total EM  $5.0 \times 10^{27}$  at temperatures from 5.5 to 7.0 dex. Solid line on left shows emission measure weighted



Figure: Response to Log-normal distributions of width 0.15 and total EM  $5.0 \times 10^{28}$  at temperatures from 5.5 to 7.0 dex. Solid lines on left shows emission measure weighted median temperature (EMWMT). 5 Spectral lines were used: Fe IX 188.497, Fe X 184.537, Fe XII 195.119, Fe XV 284.163, and Fe XVI 262.976.

# EIS Response: Narrow Distribution, Moderate EM



Assuming  $\Delta C_i \ll C_i$  (This condition can be satisfied in general by taking repeated small steps), the data corrections are given by

 $\lambda^{2} = \chi_{0}^{2} / \sum_{j} \sigma_{j}^{2} \left[ \sum_{i} \frac{C_{i} A_{ij}^{-1}}{\gamma_{i} \gamma_{j}} \right]^{2}, \quad \Delta C_{j} = \lambda \sigma_{j}^{2} \sum_{i} \frac{C_{i} A_{ij}^{-1}}{\gamma_{i} \gamma_{j}}.$  (5)

#### Iteration

This regularization step is very fast, but since the DEM has only as many basis elements as there are channels, it remains liable to producing negative emission in cases with sharp features. We therefore remove the remaining negative emission via the following iterative process:

1. Zero the negative EM in the current DEM,  $I^{(n)}$ , to create a new DEM,  $I^{(n)}_{\perp}$ 

Compute the data intensities, C<sub>i</sub><sup>+</sup> corresponding to I<sub>+</sub><sup>(n)</sup>.
 Take the difference between C<sub>i</sub><sup>+</sup> and the original C<sub>i</sub>, ΔC<sub>i</sub><sup>+</sup> = C<sub>i</sub><sup>+</sup> - C<sub>i</sub>.
 Compute correction DEM coefficients, Δi<sub>i</sub> = ∑<sub>j</sub> A<sub>ij</sub><sup>-1</sup>ΔC<sub>j</sub><sup>+</sup>/γ<sub>j</sub>.
 Subtract the corresponding DEM corrections, ΔI<sup>(n)</sup> from I<sub>+</sub><sup>(n)</sup>. By

construction, this restores I<sup>(n+1)</sup> = I<sup>(n)</sup><sub>+</sub> - ΔI<sup>(n)</sup> to agreement with the data, but reintroduces some negative emission.
6. Repeat until C<sup>+</sup><sub>i</sub> matches C<sub>i</sub> to within the desired χ<sup>2</sup>.

To speed convergence of the iteration, we also employ a linear extrapolation on  $I^{(n)}$ . The extrapolative steps are only taken when they improve  $\chi^2$ . If at first they fail, they are retried with a step size chosen so that positive components of the current DEM are reduced by no more than 150%. Performance & Summary

Detailed plots and images at right

median temperature (EMWMT).

### Narrow Distribution, Moderate EM



Figure: Response to Log-normal distributions of width 0.1 and total EM  $5.0 \times 10^{28}$  at temperatures from 5.5 to 7.0 dex. Solid lines on left shows emission measure weighted median temperature (EMWMT).

# Narrow Distribution, Higher EM



Figure: Response to Log-normal distributions of width 0.1 and total EM  $5.0 \times 10^{28}$  at temperatures from 5.5 to 7.0 dex. Solid lines on left shows emission measure weighted median temperature (EMWMT). 5 Spectral lines were used: Fe IX 188.497, Fe X 184.537, Fe XII 195.119, Fe XV 284.163, and Fe XVI 262.976.

# Active Region DEM: AIA Compared with EIS



Figure: Left: EMWMT (hue) and Total EM (intensity) from AIA DEM inversion of Active region (covered by EIS fov on April 19, 2011). Middle: Same for EIS. Right: color scale for middle and left plots.

# Loop DEM: AIA Compared with EIS



Figure: Left: Loop DEM from AIA inversion of Active region (covered by EIS fov on April 19, 2011). Right: Same for EIS.

#### Comparison with PINTofALE MCMC DEMs

- Method produces positive DEMs which match test data with good \(\chi^2\)
   Typical times for test distributions are \(\chi 0^{-3}\) seconds per pixel
   Narrow, high temperature Distributions are most difficult test cases
   Average solar DEM times are \(\chi 0.3 \times 10^{-3}\) seconds per pixel
   Further optimizations (fine-tuning, conversion to C, multi-threading) should reduce these times to microseconds
- For movies, see my ePoster (Thursday 10:10 10:50 AM) or visit http://www.physics.montana.edu/people/facdetail.asp?id\_ PersonDetails=162



Figure: Response to Log-normal distributions of width 0.1 and total EM  $5.0 \times 10^{29}$  at temperatures from 5.5 to 7.0 dex. Solid line on left shows emission measure weighted median temperature (EMWMT).

### Active Region and Coronal Loop



Figure: Middle: EMWMT (hue) and Total EM (intensity) from AIA DEM inversion of Active region (covered by EIS fov on April 19, 2011). Left: detailed DEMs for loop segment indicated by dotted lines on middle plot. No background subtraction has been performed; which part is the loop? Right: color scale for middle plot.



Figure: Comparison of DEM results (bottom) with Warren et al (ApJ 734, 90) MCMC DEM results (Top). Shown with both the selection of Iron lines mentioned above and the full selection of Warren's lines, omitting those known to be density sensitive. The DEMs matched the data with  $\chi^2$  of order unity.