

series and complex notations

$$\sum_{n=0}^{n=N} z^n = \frac{1 - z^{N+1}}{1 - z}. \quad (1)$$

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 + \dots \quad (2)$$

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 + \dots \quad (3)$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{1}{2!}x^2 + \dots \quad (4)$$

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots \quad (5)$$

$$(1+x)^p = \sum_{n=0}^{\infty} C_n^p x^n = 1 + px + \frac{p(p-1)}{2!}x^2 + \dots \quad (6)$$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!} (x-x_0)^n = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \dots \quad (7)$$

$$z = x + iy = r(\cos \theta + i \sin \theta) = re^{i\theta}. \quad (8)$$

$$\bar{z} = x - iy = r(\cos \theta - i \sin \theta) = re^{-i\theta}. \quad (9)$$

$$(e^{i\theta})^n = e^{in\theta} = \cos(n\theta) + i \sin(n\theta). \quad (10)$$

$$z^{1/n} = (re^{i\theta})^{1/n} = r^{1/n} e^{i(\theta+2k\pi)/n}, \quad k = 0, 1, 2, \dots \quad (11)$$

$$e^z = e^{x+iy} = e^x e^{iy}. \quad (12)$$

$$\sin \theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}, \quad \cos \theta = \frac{e^{i\theta} + e^{-i\theta}}{2}. \quad (13)$$

$$\sinh x = \frac{e^x - e^{-x}}{2}, \quad \cosh x = \frac{e^x + e^{-x}}{2}. \quad (14)$$

matrix operation

$$(A \pm B)_{ij} = a_{ij} \pm b_{ij}, \quad (AB)_{ij} = \sum_{k=1}^m a_{ik} b_{kj}. \quad (15)$$

$$ABC = (AB)C = A(BC), \quad (AB)^T = B^T A^T. \quad (16)$$

$$\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc, \quad \det M_{n \times n} = \sum_{(i \text{ or } j)=1}^n m_{ij} (-1)^{i+j} \det M_{(n-1) \times (n-1)}. \quad (17)$$

$$M^{-1} = \frac{C^T}{|M|}, \quad (AB)^{-1} = B^{-1} A^{-1}. \quad (18)$$

THREE USEFUL COORDINATE SYSTEMS

Cartesian Coordinates: (x, y, z)

$$d\vec{r} = dx \hat{x} + dy \hat{y} + dz \hat{z}$$

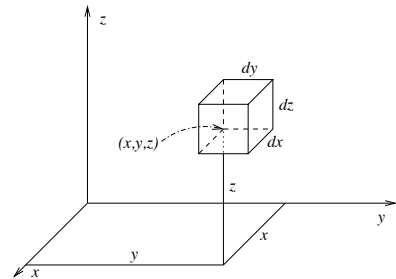
$$d\tau = dx dy dz$$

$$\vec{\nabla} f = \frac{\partial f}{\partial x} \hat{x} + \frac{\partial f}{\partial y} \hat{y} + \frac{\partial f}{\partial z} \hat{z}$$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\vec{\nabla} \cdot \vec{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

$$\vec{\nabla} \times \vec{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} \right) \hat{x} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \right) \hat{y} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \right) \hat{z}$$

**Cylindrical Coordinates:** (s, ϕ, z) $x = s \cos \phi$ $y = s \sin \phi$

$$\hat{s} = \cos \phi \hat{x} + \sin \phi \hat{y} \quad \hat{\phi} = -\sin \phi \hat{x} + \cos \phi \hat{y}$$

$$d\vec{r} = ds \hat{s} + s d\phi \hat{\phi} + dz \hat{z}$$

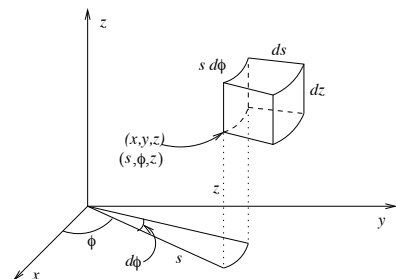
$$d\tau = s ds d\phi dz$$

$$\vec{\nabla} f = \frac{\partial f}{\partial s} \hat{s} + \frac{1}{s} \frac{\partial f}{\partial \phi} \hat{\phi} + \frac{\partial f}{\partial z} \hat{z}$$

$$\nabla^2 f = \frac{1}{s} \frac{\partial}{\partial s} \left(s \frac{\partial f}{\partial s} \right) + \frac{1}{s^2} \frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\vec{\nabla} \cdot \vec{v} = \frac{1}{s} \frac{\partial}{\partial s} (s v_s) + \frac{1}{s} \frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$$

$$\vec{\nabla} \times \vec{v} = \left[\frac{1}{s} \frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z} \right] \hat{s} + \left[\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s} \right] \hat{\phi} + \frac{1}{s} \left[\frac{\partial}{\partial s} (s v_\phi) - \frac{\partial v_s}{\partial \phi} \right] \hat{z}$$

**Spherical Coordinates:** (r, θ, ϕ) $x = r \sin \theta \cos \phi$ $y = r \sin \theta \sin \phi$ $z = r \cos \theta$

$$\hat{r} = \sin \theta \cos \phi \hat{x} + \sin \theta \sin \phi \hat{y} + \cos \theta \hat{z} \quad \hat{\theta} = \cos \theta \cos \phi \hat{x} + \cos \theta \sin \phi \hat{y} - \sin \theta \hat{z}$$

$$d\vec{r} = dr \hat{r} + r d\theta \hat{\theta} + r \sin \theta d\phi \hat{\phi}$$

$$d\tau = r^2 \sin \theta dr d\theta d\phi$$

$$\vec{\nabla} f = \frac{\partial f}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial f}{\partial \theta} \hat{\theta} + \frac{1}{r \sin \theta} \frac{\partial f}{\partial \phi} \hat{\phi}$$

$$\nabla^2 f = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial f}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial f}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 f}{\partial \phi^2}$$

$$\vec{\nabla} \cdot \vec{v} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 v_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta v_\theta) + \frac{1}{r \sin \theta} \frac{\partial v_\phi}{\partial \phi}$$

$$\vec{\nabla} \times \vec{v} = \frac{1}{r \sin \theta} \left[\frac{\partial}{\partial \theta} (\sin \theta v_\phi) - \frac{\partial v_\phi}{\partial \phi} \right] \hat{r} + \frac{1}{r} \left[\frac{1}{\sin \theta} \frac{\partial v_r}{\partial \phi} - \frac{\partial}{\partial r} (r v_\phi) \right] \hat{\theta} + \frac{1}{r} \left[\frac{\partial}{\partial r} (r v_\theta) - \frac{\partial v_r}{\partial \theta} \right] \hat{\phi}$$

