Assignment 2: due Sept 3 (Wednesday)

Problems to be submitted:

5. Do the following problems in Boas Chapter 2, about the conversion between different notations of complex numbers. Explore the shortest possible way for each problem.
Section 5, Problems 5, 15, 41, 47, 50.
Section 9, Problems 16, 18.

6. Do the following problems in Boas Chapter 2, about the convergence of a complex series.
Section 6, Problems 3, 10.
Section 7, Problems 10.

7. Boas Chapter 2, Section 10, Problem 24 – make sure to follow the instructions from (a) to (c).

Solutions:

5. [2x7 = 14 points total]
Section 5, Problem 5:

$$\left(i+\sqrt{3}\right)^2 = \left(2e^{i\frac{\pi}{6}}\right)^2 = 4e^{i\frac{\pi}{3}} = 2+2\sqrt{3}i.$$

This complex vector is in the first quadrant with absolute value 4, and phase angle 60°.

Problem 15:

$$\frac{5-2i}{5+2i} = \frac{\sqrt{29}e^{5.90i}}{\sqrt{29}e^{0.38i}} = e^{5.52i} = 0.72-0.69i.$$

This vector is in the fourth quadrant, with absolute value 1, and phase angle 5.52 (radian!).

Problem 41: each complex equation becomes two equations for the real and imaginary separately. In this problem, we get two algebra equations $2x-3y-5=0$, and $x+2y+1=0$; solving them, we find $x=1, y=-1$.

Problem 47: following what we have done in the class, we can find the cube root of -1, so the equation becomes

$$x+iy = (-1)^{\frac{1}{3}} = e^{i(\pi+2k\pi)/3}.$$

This gives us 3 solutions for $k=0,1,2$ respectively:

$$x+iy = e^{i\frac{\pi}{3}},$$

$$x+iy = e^{i\pi},$$

$$x+iy = e^{i\frac{5\pi}{3}}.$$

Therefore, we find 3 sets of solutions of (x, y): $(1/2, \sqrt{3}/2)$, $(-1, 0)$, and $(1/2, -\sqrt{3}/2)$.

Problem 50: the equation is written as $\sqrt{x^2+y^2} = y - ix$. It is seen that the imaginary part $x = 0$, and the real part $\sqrt{x^2+y^2} = y$, so y can be any non-negative real number.

Section 9, problem 16:

$$(i - \sqrt{3})(1 + i\sqrt{3}) = -2\sqrt{3} - 2i.$$

For this problem, either way (rectangular or Euler notion) is easy.

Section 9, problem 18:

$$\left(\frac{1+i}{1-i}\right)^4 = \left(\frac{e^{i\frac{\pi}{4}}}{e^{i\frac{7\pi}{4}}}\right)^4 = e^{-i6\pi} = 1.$$

For this problem, using Euler notation is apparently easier for the computation, which can be then converted back to rectangular notation.

6. [1+1+2 points]
Section 6, Problem 3: we can use the ratio test,

$$\rho = \frac{1}{|1+i|} = \frac{1}{\sqrt{2}} < 1,$$

so the series converges.

Section 6, Problem 10: again we use the ratio test,

$$\rho = \frac{|1+i|}{|1-i\sqrt{3}|} = \frac{\sqrt{2}}{2} < 1,$$

so the series converges.

Section 7, Problem 10: we use the ratio test,

$$\rho_n = |iz|\frac{n}{n+1},$$

as $n \to \infty$, $\rho \to |z|$. For the series to converge, it is required $|z| = \sqrt{x^2+y^2} < 1$, so the convergece disk is a circle centered at origin with radius $r = 1$ in the complex plane.

7. [4 points] Section 10, Problem 24: we will re-write the expression into the Euler formula as

$$\left(e^{i\left(\frac{4}{3}\pi + 2k\pi\right)}\right)^{\frac{1}{8}},$$

which has 8 solutions for $k = 0, 1, ...7$. In Euler notation, these 8 solutions are $e^{i\frac{1}{6}\pi}$, $e^{i\frac{5}{12}\pi}$, $e^{i\frac{2}{3}\pi}$, $e^{i\frac{11}{12}\pi}$, $e^{i\frac{7}{6}\pi}$, $e^{i\frac{17}{12}\pi}$, $e^{i\frac{5}{3}\pi}$, and $e^{i\frac{23}{12}\pi}$. The modulus of these solutions is 1, and they divide the complex space into 8 parts, separated by $\frac{\pi}{4}$, or 45° between adjacent solutions, as shown in the figure.