

Physics 425, Spring 2012

Assignment set #6

Due February 15:

13. In this problem, you will study wave dispersion in a dielectric insulating media whose electric susceptibility χ_E is a complex function of the EM wave frequency ω as:

$$\chi_E = \frac{q^2}{m\epsilon_0} \frac{1}{(\omega_0^2 - \omega^2 - i\gamma\omega)}$$

where q , m , ω_0 , γ are constants determined by the properties of the material, and ϵ_0 is permittivity in vacuum. For a plane EM wave in this media, its wave number can be written in the familiar way: $k = \omega\sqrt{\mu_0\epsilon}$. Now that ϵ is a complex number, the wave number k is also a complex number that may be written as $k = k_1 + ik_2$, with k_1 and k_2 being real numbers.

- (a) write down the expression of the electric field in the plane wave traveling in z direction; in your solution, you should note the damping term and the oscillation term.
- (b) If $|\chi_E| \ll 1$, find the phase speed of the wave, and the “skin” depth for the damping term, namely, the distance at which the electric field is diminished to $1/e$.

Due February 17:

14. Griffiths 9.25

Due February 22:

15. In the class, we discussed that the displacement of charges under the impact of EM waves of form $E = E_0 e^{-i\omega t}$ is given by

$x = \frac{q}{m} \frac{1}{(\omega_0^2 - \omega^2) - i\gamma\omega} E$. Now we use this formula to discuss wave

dispersion in conducting plasmas. In the plasma, we may consider that the restoring force vanishes, i.e., $\omega_0 = 0$. The plasma may carry a current given by $J = Nq \frac{dx}{dt}$, where N is the number density of charges (for example, electrons).

(a) Use Ohm's law to find the conductivity of the plasma.

(b) In a dilute plasma, the dissipation term also vanishes, i.e., $\gamma = 0$.

Find the conductivity in this case. Is there a phase shift between the electric field and electric current? What is the amount of the phase shift?

(c) Now take the conductivity derived in (b) to the dispersion relation given by Equation 9.124 (pp. 394) in the textbook, find the wave number k for an EM wave in the plasma. Discuss in what condition the wave will be attenuated.

(d) In what condition will the wave propagate with no attenuation? Find the phase velocity and group velocity of the propagating wave.