

## Physics 425, Spring 2012

### Assignment set #7

Due February 24:

16. The electromagnetic fields between two parallel conducting plates are given as  $\vec{E} = E_0 e^{i(kz - \omega t)} \hat{x}$ ,  $\vec{B} = \frac{E_0}{c} e^{i(kz - \omega t)} \hat{y}$ . Now consider that we roll the two plates into infinitely long co-axial cables, with the inner radius  $a$  and outer radius  $b$ . We still let the EM waves traveling between the two shells in the  $z$  direction along the axis of the cable. In this case, the electric field becomes radial pointing from one shell to the other, and the magnetic field becomes azimuthal around the cable axis. So we shall write the fields as  $\vec{E} = E(s) e^{i(kz - \omega t)} \hat{s}$ ,  $\vec{B} = B(s) e^{i(kz - \omega t)} \hat{\phi}$ .
- Using Gauss's law  $\vec{\nabla} \cdot \vec{E} = 0$  in cylindrical coordinates, show that the expression of  $E(s)$  is given by  $E(s) = \frac{A}{s}$ , where  $A$  is a constant.
  - Using Faraday's law in cylindrical coordinates, find the magnetic field  $B(s)$ .
  - Using the boundary condition  $(\epsilon_{out} \vec{E}_{out} - \epsilon_{in} \vec{E}_{in}) \cdot \hat{n} = \sigma_f$ , find the surface charge on the inner and outer shells of the cable. At a given time (e.g.  $t = 0$ ), what is the surface charge like along the cable?
  - using the boundary condition:  $\vec{B}_{out} / \mu_{out} - \vec{B}_{in} / \mu_{in} = \vec{K}_f \times \hat{n}$ , find the surface current on the inner and outer shells of the cable. At a given time (e.g.  $t = 0$ ), what is the surface current like along the cable?

Due February 27:

17. Coaxial cable transmission line: in the class, we've got the solutions of electromagnetic fields between the two cylindrical conducting shells, and surface charge and current densities at the inner and outer conducting surfaces.

(a) Show that the solutions of surface charge  $\sigma$  and the surface current density  $K$  satisfy the continuity equation  $\vec{\nabla} \cdot \vec{K} + \frac{\partial \sigma}{\partial t} = 0$  (this is the form of the more familiar equation  $\vec{\nabla} \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$  for surface charge and current). You should not be surprised, since by abiding by the continuity equation, Maxwell introduced the displacement current, which leads to the wave solutions in the first place.

(b) From the solution of  $\vec{B} \equiv B_\phi \hat{\phi}$  derived in the class, use the integral form of Faraday's law to find the  $z$ -component of the induced electric field  $E_{in,z}$  between the two conducting shells ( $a < s < b$ ), assuming that the field at the outer surface ( $s = b$ ) vanishes and ignoring the radial component of the field.

(c) From the solution of the electric potential  $V$  derived in the class, find the  $z$ -component of the electrostatic field  $E_{st,z}$  by  $\vec{E} = -\vec{\nabla}V$  at the conducting surfaces.

(d) From (b) and (c), Prove that at the inner boundary ( $s = a$ ),  $E_z = E_{in,z} + E_{st,z} = 0$ . It has to be so for a perfect conductor, since  $\vec{E} = \frac{\vec{J}}{\sigma}$  (here  $\sigma$  is the conductivity, not surface charge density), where  $\sigma = \infty$  for a perfect conductor.