

Solutions to Quiz 1

(a) The wave propagates in $+z$ direction. The electric field vector has an inclination of 45° with the x -axis. The wave is linearly polarized since the x and y components are in phase, so that the angle between the electric field and the x -axis is a constant.

(b) From the Faraday's law $\vec{\nabla} \times \vec{E} = -\partial\vec{B}/\partial t$, we get the relation $\vec{k} \times \vec{E} = \omega\vec{B}$. Note that even though this relation is derived with the complex wave solution, it also applies to the real part of the complex solution, which is the wave solution given here. In this problem, $\vec{k} = k\hat{z}$, so $\vec{B} = \frac{k}{\omega}\hat{z} \times \vec{E} = \frac{1}{c}\frac{E_0}{\sqrt{2}}\cos(kz - \omega t)(-\hat{x} + \hat{y})$.

(c) The light intensity $I = \langle S \rangle$, where S is the magnitude of Poynting flux given by $\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B} = c\epsilon_0 E_0^2 \cos^2(kz - \omega t)\hat{z}$. So $I = \langle S \rangle = \frac{1}{2}c\epsilon_0 E_0^2$, leading to $E_0 = \sqrt{\frac{2}{c\epsilon_0}} \approx 27$ V/m, and the amplitude of the magnetic field is $B_0 = E_0/c \approx 9 \times 10^{-8}$ Tesla.

(d) In this case, the light beam is entirely reflected. We know that in all conditions, the tangential component of the electric field is continuous. Also in the configuration, the electric field at the interface only has a tangential component (which includes both the \hat{x} and \hat{y} components!). So the continuity condition $\vec{E}_{||}^1 = \vec{E}_{||}^2$ is directly written as $E_{I0} + E_{R0} = E_{T0} = 0$, or $E_{R0} = -E_{I0}$ at the boundary. Therefore, the electric field of the reflected wave is $\vec{E}_R = -\frac{1}{\sqrt{2}}E_0\cos(-kz - \omega t)(\hat{x} + \hat{y})$.

(e) The magnetic field at the boundary only also has the tangential component, but note that in this problem there can be free current at the metal surface. In this case, we simply use Faraday's law to find the magnetic field by $\vec{B}_R = \frac{1}{\omega}(-\vec{k}) \times \vec{E}_R = -\frac{k}{\omega}\hat{z} \times \vec{E}_R = \frac{1}{c}\frac{E_0}{\sqrt{2}}\cos(-kz - \omega t)(-\hat{x} + \hat{y})$.