

Solutions to Quiz 2

(a) From the law of reflection and transmission, $\sin(\theta_I)/\sin(\theta_T) = n_2/n_1$. In this case, $n_1 = 1, n_2 = 1.414$, $\sin(\theta_T) = \sin(30^\circ) = 0.5$, so $\sin(\theta_I) = 1.414 \times 0.5 = \frac{\sqrt{2}}{2}$, leading to $\theta_I = 45^\circ$. So the sun observed by David is 45° from zenith.

(b) Upon the water surface, the incident light intensity measured by David is $I_I = I \cos\theta_I = 1400 * 0.707 = 1000\text{W/m}^2$. The transmitted light intensity is given by

$$\frac{I_T}{I_I} \equiv T = \alpha\beta\left(\frac{2}{1+\alpha\beta}\right)^2$$

This is for EM waves polarized in the direction normal to the incidence plane. In this problem, $\alpha = \cos(\theta_T)/\cos(\theta_I) = \sqrt{3}/\sqrt{2}$, $\beta = n_2/n_1 = \sqrt{2}$, so $\alpha\beta = \sqrt{3}$, and $T = 4\sqrt{3}/(1+\sqrt{3})^2 = 0.93$, and $I_T = 0.93I_I = 930\text{W/m}^2$.

(c) The wavelength is given by $\lambda = 2\pi/k_+$. In this case, the fluid can be considered as a good conductor, so

$$k_+ \approx \omega\sqrt{\epsilon\mu}\sqrt{\frac{\sigma}{2\epsilon\omega}}$$

One can see that the wave number is much larger than that in the air, so the wavelength in the fluid becomes much smaller than that in the air. The wave number in the air is $k_0 = \omega\sqrt{\epsilon_0\mu_0}$. These two are different by:

$$\frac{k_+}{k_0} = \sqrt{\frac{\epsilon}{\epsilon_0}}\sqrt{\frac{\sigma}{2\epsilon\omega}} = 1.414\sqrt{5} = 3.2$$

So the wavelength in the fluid is smaller by about a factor of 3.

(d) The transmitted electric field in the fluid should be written as:

$$\vec{E} = E_0 e^{i\Phi_E} e^{-k_- \cdot \vec{r}} e^{i(k_+ \cdot \vec{r} - \omega t)} \hat{y}$$

In this case, the wave vector $\vec{k} = k \sin(\theta_T) \hat{x} + k \cos(\theta_T) \hat{z}$, where $k = k_+ + ik_-$. So the $\vec{k} \cdot \vec{r}$ term becomes $k \sin(\theta_T)x + k \cos(\theta_T)z$, and the electric field is written as:

$$\vec{E} = E_0 e^{i\Phi_E} e^{-k_- [\sin(\theta_T)x + \cos(\theta_T)z]} e^{i[k_+ \sin(\theta_T)x + k_+ \cos(\theta_T)z - \omega t]} \hat{y}$$

Using Faraday's law, we find the magnetic field to be:

$$\vec{B} = \frac{1}{\omega} \vec{k} \times \vec{E} = \frac{|k|}{\omega} e^{i\Phi_k} E_0 e^{i\Phi_E} e^{-k_- [\sin(\theta_T)x + \cos(\theta_T)z]} e^{i[k_+ \sin(\theta_T)x + k_+ \cos(\theta_T)z - \omega t]} [\sin(\theta_T) \hat{z} - \cos(\theta_T) \hat{x}]$$

Here $|k| = \sqrt{k_+^2 + k_-^2} \approx \omega\sqrt{\epsilon\mu}\sqrt{\frac{\sigma}{\epsilon\omega}} \approx \sqrt{10}\omega\sqrt{\epsilon\mu}$, and phase angle $\Phi_k = \tan^{-1}\left(\frac{k_-}{k_+}\right) \approx 45^\circ$.

The Poynting flux is computed using only the real part of the fields as

$$\vec{S} = \frac{1}{\mu} \vec{E} \times \vec{B}$$

$$\begin{aligned}
 &= \frac{|k|E_0^2}{\mu\omega} e^{-2k_-[\sin(\theta_T)x + \cos(\theta_T)z]} \cos[k_+ \sin(\theta_T)x + k_+ \cos(\theta_T)z - \omega t + \Phi_E] \\
 &\quad \times \cos[k_+ \sin(\theta_T)x + k_+ \cos(\theta_T)z - \omega t + \Phi_E + \Phi_k] [\sin(\theta_T)\hat{x} + \cos(\theta_T)\hat{z}]
 \end{aligned}$$

$|k|$ and Φ_k are as given above.

Note that the above expressions appear complicated, but the main points are: (a) both the electric field and magnetic field have an oscillation term (by $\vec{k}_+ \cdot \vec{r}$) and a damping term (by $\vec{k}_- \cdot \vec{r}$); (b) the magnetic field has a phase shift Φ_k with respect to the electric field; (c) the Poyning flux should be calculated using the real parts of the electric and magnetic field expressions.