

Solutions to Quiz 3

(a) From the boundary condition, the surface charge is given by $\sigma_f = -\epsilon_0 \hat{n} \cdot \vec{E}$ since the electric field \vec{E}_{out} in the conductor is zero. At $x = 0$, $\hat{n} = -\hat{x}$, and at $x = d$, $\hat{n} = \hat{x}$, and the electric field between the ribbons is normal to the surface $\vec{E} = E_x \hat{x}$. Therefore, the surface charge density at the two ribbons are:

$$\begin{aligned}\sigma(x = 0) &= \epsilon_0 E_0 e^{i(kz - \omega t)} \\ \sigma(x = d) &= -\epsilon_0 E_0 e^{i(kz - \omega t)}\end{aligned}$$

From Farady's law, for plane wave solution, the magnetic field is

$$\vec{B} = \frac{1}{c} \hat{k} \times \vec{E} = \frac{E_0}{c} e^{i(kz - \omega t)} \hat{y}$$

Note that \vec{k} , \vec{E} , and \vec{B} make the right-hand triangle. The surface current density is given by $\vec{K}_f = -\hat{n} \times \vec{B} / \mu_0$, which leads to:

$$\begin{aligned}\vec{K}(x = 0) &= \frac{E_0}{\mu_0 c} e^{i(kz - \omega t)} \hat{z} \\ \vec{K}(x = d) &= -\frac{E_0}{\mu_0 c} e^{i(kz - \omega t)} \hat{z}\end{aligned}$$

These solutions are consistent with the direction of the electric field pointing from positive to negative charges and the magnetic field following the right hand rule.

(b) The electric potential difference is

$$V = - \int \vec{E} \cdot d\vec{l} = E_0 d e^{i(kz - \omega t)}$$

The total current along the ribbon is

$$I = K b = \frac{E_0 b}{\mu_0 c} e^{i(kz - \omega t)}$$

The impedance is therefore

$$Z = \frac{V}{I} = \frac{\mu_0 c d}{b}$$

(c) From the dispersion relation

$$k^2 = \mu_0 \epsilon_0 \omega^2 \left(1 - \frac{\gamma}{\omega}\right) = \frac{\omega^2}{c^2} \left(1 - \frac{\gamma}{\omega}\right)$$

we see that if $k^2 < 0$, k becomes an imaginary number, so the wave will be attenuated. That requires

$$\left(1 - \frac{\gamma}{\omega}\right) < 0$$

or $\omega < \gamma$. So waves with frequency lower than γ will be attenuated. The magnetic field in the plane wave solution

$$\vec{B} = \frac{k}{\omega} \hat{k} \times \vec{E}$$

When k is an imaginary number, \vec{B} is phase shifted by $\pi/2$ relative to the electric field.

(d) When $k^2 > 0$, or $\omega > \gamma$, k is a real number so wave propagates. From the dispersion relation, we find:

$$2kdk = 1/c^2(2\omega - \gamma)d\omega$$

leading to the group velocity

$$\frac{d\omega}{dk} = \frac{kc^2}{\omega(1 - \gamma/2\omega)} = c \frac{\sqrt{1 - \gamma/\omega}}{1 - \gamma/2\omega}$$