

Solutions to Quiz 4

(a) The scalar potential is given by

$$V(\vec{r}, t) = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda dl'}{\iota}$$

Since the line charge is lying on the y-axis, and taking into account the retarded potential, we can further write the above as:

$$V(\vec{r}, t) = \frac{1}{4\pi\epsilon_0} \int \frac{ky' \cos[\omega(t - \iota/c)] dy'}{\iota}$$

where $\iota = [x^2 + (y - y')^2 + z^2]^{1/2}$.

(b) The direction of the dipole moment is \hat{y} .

(c) The dipole moment is derived as

$$\vec{p} = \int \lambda \vec{r}' dy' = k \cos(\omega t) \int_{-d}^d y'^2 dy' \hat{y} = \frac{2}{3} k d^3 \cos(\omega t) \hat{y} = p_0 \cos(\omega t) \hat{y}$$

where we take $p_0 = \frac{2}{3} k d^3$.

(d) The magnetic field by the dipole is given by

$$\vec{B} = \vec{\nabla} \times \vec{A} = \frac{\mu_0}{4\pi} \vec{\nabla} \times \frac{\partial \vec{p}}{r}$$

For this solution, we will disregard higher order terms, or $1/r^2$ terms, so the solution can be simplified as:

$$\vec{B} = \frac{\mu_0}{4\pi} \left(-\frac{1}{c}\right) \hat{r} \times \frac{\partial^2 \vec{p}}{\partial t^2} = -\frac{\mu_0}{4\pi c r} \frac{\partial^2 p}{\partial t^2} (\hat{r} \times \hat{y})$$

Use the spherical coordinates, $\hat{y} = \sin \theta \sin \phi \hat{r} + \cos \theta \sin \phi \hat{\theta} + \cos \phi \hat{\phi}$, the above solution becomes

$$\vec{B} = \frac{\mu_0}{4\pi c r} \frac{\partial^2 p}{\partial t^2} (-\cos \theta \sin \phi \hat{\theta} + \cos \phi \hat{\phi})$$

The magnetic field oscillates in the spherical plane. The electric field is orthogonal to the magnetic field and to \hat{r} as well, and its magnitude is c times the magnitude of the magnetic field, so it can be directly written as

$$\vec{E} = \frac{\mu_0}{4\pi} \frac{1}{r} \frac{\partial^2 p}{\partial t^2} (-\cos \theta \sin \phi \hat{\theta} - \cos \phi \hat{\phi})$$

where $\partial^2 p / \partial t^2 = -\omega^2 p_0 \cos(\omega\tau)$, with $\tau \equiv t - r/c$.

(e) The Poynting flux is always in \hat{r} direction. The magnitude of the Poynting flux is proportional to $1/r^2$ and $\sin^2 \psi$, where r is the distance and ψ is the angle between the dipole moment and the observer's position. The angle between the dipole moment and the first observer is $\psi_1 = 90^\circ$, and $\psi_2 = 30^\circ$. Therefore, we can see that

$$\frac{S_1}{S_2} = \frac{r_2^2 \sin^2 \psi_1}{r_1^2 \sin^2 \psi_2} = 4 \times \frac{1}{1/4} = 16$$