

Solutions to Quiz 6

(a) The particle's proper time is shorter by a factor of $\gamma_u = 1/\sqrt{1 - u^2/c^2} = \sqrt{2}$. Therefore, $u = \sqrt{2}/2c$. In S frame, $u_x = 0$, $u_y = \sqrt{2}/2c$, and $u_z = 0$.

(b) We can first calculate the velocity of the particle in the S' frame. Here $\gamma = 1/\sqrt{1 - v^2/c^2} = \sqrt{2}$.

$$u'_x = \frac{u_x - v}{1 - u_x v/c^2} = -v = -\sqrt{2}/2c$$

$$u'_y = \frac{u_y}{\gamma(1 - u_x v/c^2)} = \frac{u_y}{\gamma} = \frac{1}{2}c$$

$$u'_z = \frac{u_z}{\gamma(1 - u_x v/c^2)} = 0$$

So $\gamma_{u'} = 1/\sqrt{1 - u'^2/c^2} = 2$. The total energy is $E' = \gamma_{u'} mc^2 = 2mc^2$. The relativistic momentum is:

$$p'_x = \gamma_{u'} m u'_x = -\sqrt{2}mc$$

$$p'_y = \gamma_{u'} m u'_y = mc$$

$$p'_z = \gamma_{u'} m u'_z = 0$$

We can also find this in a different way. The 4-element relativistic momentum in S frame is given by

$$p^\mu = (\gamma_u mc, \gamma_u m u_x, \gamma_u m u_y, \gamma_u m u_z) = (\sqrt{2}mc, 0, mc, 0)$$

The 4-element relativistic momentum in S' frame is derived by Lorentz transformation:

$$p'^0 = \gamma p^0 - \beta \gamma p^1 = \gamma p^0 = 2mc$$

$$p'^1 = \gamma p^1 - \beta \gamma p^0 = -\beta \gamma p^0 = -\sqrt{2}/2 \sqrt{2} \sqrt{2} mc = -\sqrt{2}mc$$

$$p'^2 = p^2 = mc$$

$$p'^3 = p^3 = 0$$

The first element gives $E' = p'^0 c = 2mc^2$, and the other three elements are relativistic momentum. These are the same as derived above.

(c) In the particle's rest frame, the magnetic field becomes $B'_x = \gamma_u B_x = \sqrt{2}B$. The particle also feels an electric field as $\vec{E}' = \gamma_u \vec{u} \times \vec{B} = -\sqrt{2}\sqrt{2}/2c B \hat{z} = -cB \hat{z}$. The field is in $-z$ direction.

(d) The total momentum of the positron and electron of the same speed in a head-on collision is zero, so the two photons produced in this process must carry the same amount of momentum but of opposite signs to observe the momentum conservation. Since photon energy is $E_{ph} = pc$, so the two photons have the same energies. From the energy conservation, we have $2E_{ph} = 2E_e$, or $E_{ph} = E_e = 2m_e c^2$.