

## Solutions to Homework Problem Set 4

4.1 The average thermal speed of a hydrogen atom on the Sun is

$$v_{th} = \sqrt{\frac{3kT}{m_H}} = \sqrt{\frac{3 \times 1.38 \times 10^{-23} \text{ J/K} \times 5800 \text{ K}}{1.673 \times 10^{-27} \text{ kg}}} = 12 \text{ km/s}$$

key: average thermal speed

4.2 (a) The escape speed from the Sun's surface is given by

$$v_{esc} = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \text{ Nm}^2/\text{Kg}^2 \times 1.989 \times 10^{30} \text{ kg}}{6.96 \times 10^8 \text{ m}}} = 617 \text{ km/s}$$

(b) Using the thumb rule, it is seen that  $6v_{th} = 72 \text{ km/s} \ll 617 \text{ km/s} = v_{esc}$ , so hydrogen atoms cannot escape the Sun's gravity.

key: escape speed

4.3 Key point: since Titan and Ganymede have comparable mass and radius, the escape speed from the surface of Titan is also comparable with the escape speed from the surface of Ganymede. On the other hand, Ganymede is closer to the Sun with a slightly higher surface temperature than Titan, thus is less capable of retaining gases. But more importantly, the current theory goes that Titan's atmosphere consisting primarily of nitrogen molecules may be a result of outgassing from below its surface, while such volatile process is not present on Ganymede to produce an appreciable atmosphere.

4.4 (a) The object orbits around the Sun hence we can apply Kepler's third law  $P^2 = a^3$ . Here,  $a$  is the distance from the Sun in AU and  $P$  is the orbital period in Earth's year. The orbital period is then calculated as  $P = \sqrt{a^3} = \sqrt{100^3} = 1000 \text{ yr}$ .

(b) It takes 1000 years for the object to finish one orbit, or 360 degrees in terms of angular distance, then to move 1 arcminute across the sky, it takes  $t = \frac{1000 \times 365 \text{ days}}{360 \times 60 \text{ arcmin}} \approx 17 \text{ days/arcmin}$ .

(c) From (b), it is seen that these objects move very slowly across the sky, thus it needs patience to detect their motions. (d) At large distances, these objects will have very small angular size, which is inversely proportional to the distance. They are also very dim, because the apparent brightness decreases with the distance squared. Large telescopes have large light gathering power (proportional to the aperture squared) to discover faint objects, and also have better angular resolution (inversely proportional to the aperture) to resolve small objects.

Key: Kepler's third law; angular measurements; angular resolution and light gathering power of telescopes.

4.5 Key points: Plate tectonic is driven by its internal heat giving rise to convection motion in the aethenosphere, which pushes upper thin and cool crustal plates into motion. Upwelling hot molten materials from the mantle force ocean plates open and to move apart and collide with or rub against other plates. Where plates separate, new materials from below form ocean ridges. Where plates collide, the heavy ocean plate sink below the thin continental plate, or subduction, so crustal materials are recycled back to the mantle, and deep ocean trenches and mountain ranges are formed.

Geographic features generated by plate tectonics include earthquake epicenter belts (e.g., in California), ocean rifts (e.g. Mid-Atlantic ridge), gulfs (e.g. Suez Gulf), trenches (e.g. Peru-Chile Trench), mountain ranges (e.g., Himalayas) at the boundaries of plates, where plates separate, collide or rub against each other (see the textbook and lecture notes for more examples).

4.6 (a) The Sun’s radiation flux at the surface can be calculated using Stefan-Boltzmann’s law, and the total radiation power is the integral of the radiation flux across the entire surface of the sphere. The entire radiation power spreads out isotropically, and the radiation flux (i.e., power per unit area) received by an imaginary surface at the distance  $d$  is the total radiation power divided by the area of a sphere whose radius is  $d$ . This is written as

$$F = \frac{\sigma T_{sun}^4 \times 4\pi R_{sun}^2}{4\pi d^2} = \sigma T_{sun}^4 \frac{R_{sun}^2}{d^2}$$

In other words, the radiation flux decreases with the distance squared. At Earth,  $d = 1 \text{ AU} = 1.5 \times 10^{11} \text{ m}$ , so the solar constant is

$$F = \sigma T_{sun}^4 \frac{R_{sun}^2}{d^2} = 5.67 \times 10^{-8} \text{ W/K}^4/\text{m}^2 \times (5800 \text{ K})^4 \times \left(\frac{7 \times 10^8 \text{ m}}{1.5 \times 10^{11} \text{ m}}\right)^2 = 1400 \text{ W/m}^2$$

The solar radiation flux arriving at the top of Earth’s atmosphere is about  $1400 \text{ W/m}^2$ .

(b) The total amount of solar radiation power absorbed by Earth is given by  $P_{in} = F(1 - A)\pi R_{earth}^2$  where  $\pi R_{earth}^2$  is the effective receiving area of the hemisphere facing the Sun, and  $A$  is the albedo of Earth, or the fraction of sunlight reflected by Earth. Earth also radiates as a blackbody at its surface temperature, which can be written as  $P_{out} = 4\pi R_{earth}^2 \sigma T_{earth}^4$ . To maintain thermal equilibrium, Earth radiates the same amount of power as it receives from the Sun, hence  $P_{in} = P_{out}$ . Therefore, Earth’s mean surface temperature can be calculated by  $F(1 - A)\pi R_{earth}^2 = 4\pi R_{earth}^2 \sigma T_{earth}^4$ , or

$$T_{earth}^4 = \frac{F(1 - A)}{4\sigma} = \frac{1400 \text{ W/m}^2(1 - 0.31)}{4 \times 5.67 \times 10^{-8} \text{ W/m}^2/\text{K}^4} = 4.25 \times 10^9 \text{ K}^4$$

and  $T_{earth} = 255 \text{ K}$ . This is the theoretical mean surface temperature of Earth.

(c) The measured mean surface temperature of Earth is  $287 \text{ K}$ , about  $30 \text{ K}$  higher than the theoretical value. This difference is significant, because  $255 \text{ K}$  is  $18 \text{ K}$  below the water freezing point, and  $287 \text{ K}$  is  $14 \text{ K}$  above the freezing point. If Earth’s temperature were  $255 \text{ K}$ , there would not be

stable existence of large body of liquid water, and life might not have occurred on Earth. However, thanks to the natural greenhouse effect, Earth is much warmer to contain liquid water, which is believed to be crucial for life.