

Solutions to Homework Problem Set 9

Chap 13, Q51: When Titan is moved to where the Earth's moon is, its temperature will rise significantly due to the close distance to the Sun. On the other hand, the escape speed on Titan is similar to that on the Moon. As a rough estimate, we may take the daytime temperature of the Moon to find that six times the thermal speeds of molecular nitrogen and methane gases in Titan's atmosphere will exceed the escape speed on Titan. Therefore, Titan would not be able to retain its atmosphere any more and would gradually become airless.

As Titan is larger than the Moon by a factor of 1.5 (in diameter), its angular size would be larger by the same factor when placed at the Moon's orbit and cast a larger shadow on the Earth when aligned between the Sun and the Earth. Therefore there would be more chances to see total and partial solar eclipses, but there would not be annular solar eclipses as the angular size of the Titan, even at aphelion, would be larger than the angular size of the Sun.

Chap 15, Q8: Asteroids whose orbital periods are simple fractions of Jupiter's orbital period are regularly aligned between Jupiter and the Sun, and as time goes on, the resonance effect of Jupiter's gravity gradually deflects these asteroids from their original orbits, thus forming the Kirkwood gaps in the asteroid belt. The gaps in Saturn's rings formed in very similar way, in that ring particles at certain orbits are deflected by the resonance effect of the gravity of Saturn's satellites.

Chap 15, Q24: The dust tail consists of neutral dust particles. When a comet is approaching the Sun, dust particles are pushed away from the Sun by the Sun's radiation pressure while still trying to retain the original trajectory along the comet's orbit. The ion tail is composed of charged particles, which interact with the Sun's magnetic field in solar wind to form a relatively straight ion tail pointing away from the Sun. These two different mechanisms produce dust and ion tails with different directions.

A1: According to Newton's law of universal gravitation, the orbital motion of Earth or any object on Earth's orbit around the Sun is maintained by the gravitational force of the Sun, or the Sun's gravitational force provides the centripetal force for the circular motion in such a way $f_G = F_G/m = \omega^2 r_E$ where $f_G = F_G/m$ is Sun's gravitational force per unit mass on Earth's orbit, $\omega = 2\pi/P$ is the angular speed of Earth's orbit, P being the orbital period, and r_E is Earth's distance to Sun. Now consider a small body at L1: it would receive the gravitational forces by both Sun and Earth pointing in opposition directions. The net gravitational force on the small body would be smaller than the gravitational force only applied by Sun. This net gravitational force provides the centripetal force for the small body to orbit Sun, which is written as $f'_G = \omega'^2 r'$. From the above argument, it is seen that $f'_G < f_G$. Therefore, we can find a particular distance $r' \equiv r_{L1} < r_E$, at which the small body under the gravities of Sun and Earth, orbits at the same angular speed $\omega' = \omega$ as Earth. Note that in the discussion we regard that the gravitational force by the small body on Earth is too small and ignored.

In the same way, we can see that at $L2$, the configuration of Sun, Earth, and the small body is such that the small body would receive gravitational forces from Sun and Earth, both pointing toward the Sun. Therefore, the combined force makes a larger centripetal force than just the Sun's gravitational force. With the larger f'_G , the small body can orbit the Sun at a larger distance $r' \equiv r_{L2} > r_E$ while keeping the same angular speed $\omega' = \omega$ as Earth.