

## Physics 317, Fall 2009

Problem set # 9, week of October 26

Due October 28:

38. Two parallel conducting plates make a capacitor and are charged with  $Q$  and  $-Q$ , respectively. The area of each plate is  $A$ . The distance  $d$  between the plates is small compared with the size of the plates, so that end effects can be ignored. We can compute the energy stored in the capacitor in the following different ways.

(a) Find the electric field generated by this configuration, and compute the electric energy

by  $W = \int_{\text{all space}} u_E d\tau$ , where  $u_E = \frac{\epsilon_0 E^2}{2}$  is the electric field energy density.

(b) Imagine that one plate is pulled by an external force against the electrostatic force by the other plate to move away from the other plate to a distance  $d$ . How much work is done by this imaginary external force? The amount of work is equivalent to the energy of the capacitor. (You may already find that the expression of the electrostatic pressure is the same as the electric field energy density!)

(c) You may compute the energy of one conductor by  $W_1 = \frac{1}{2} \int \sigma_1 V_1 da_1$ , using an arbitrary point as the reference point for  $V_1$ . The total energy is the sum of the energy of the two conductors. You will find that the choice of the reference point does not matter.

(d) We may consider the work done to charge up a capacitor. In the intermediate stage, the capacitor has the charge  $q$ , and the potential difference between the plates is  $V=q/C$ , where  $C$  is the capacitance, a constant only determined by the geometry of the capacitor. To charge up the capacitor by an amount of charge  $dq$  at this stage, work done is  $dW = Vdq = qdq/C$ . Compute the total work done to charge the capacitor from  $q = 0$  to  $q = Q$ , which is the energy stored in the capacitor.

Due October 30:

39. In this problem, you are going to solve 1-dimensional Laplace's equations to find the electric potential.

(a) Consider two large parallel metal plates. One is placed on the  $xy$  plane and grounded with a constant potential  $V_0$ . The other plate is placed a distance  $d$  above the first plate, and is uniformly charged with positive charge density  $\sigma$ . Find the electric potential  $V(z)$  between the two plates. Note that you should write down the boundary condition in this problem as  $V(0) = V_0$  and  $\frac{\partial V}{\partial z}|_{z=d} = E(d)$ , where  $E(d)$  should be determined by the

boundary condition  $\vec{E}_{above} - \vec{E}_{below} = \frac{\sigma}{\epsilon_0} \hat{n}$ .

(b) Find the electric potential by a metal sphere of radius  $R$  carrying charge  $Q$ . Here you shall write down the Laplace's equation in spherical coordinates. The boundary conditions are: at infinity,  $V = 0$ , and at  $r = R$ ,  $\vec{E}(R) = -\nabla V|_{r=R}$ , where  $\vec{E}(R)$  is determined using the boundary condition  $\vec{E}_{above} - \vec{E}_{below} = \frac{\sigma}{\epsilon_0} \hat{n}$ .

Due November 2:

40. Griffiths Problem 3.13

41. Griffiths Problem 3.14