

## Part II: Magnetic fields in matter (Chap6)

### II.1 Torques and forces on magnetic dipoles

An external magnetic field may exert a torque on a magnetic dipole:  $\vec{N} = \vec{m} \times \vec{B}$ . The nature of it is the torque by the Lorentz force on a current loop). The effect of the torque tends to rotate the dipole into such an orientation that  $\vec{m} \parallel \vec{B}$ , or the equilibrium orientation. At this orientation,  $\vec{N} = 0$ , and the magnetic energy of the dipole (at that same location)  $U_m = -\vec{m} \cdot \vec{B}$  is minimized.

Example 1: magnetic torque on a square loop current (see book).

### II.2 Magnetization and magnetic field by magnetized materials

Some materials may be magnetized by an external field. The magnetization may be envisioned as line-up of miniature magnetic dipoles on atomic scales. Inside the material, spinning and orbiting charges (most significantly electrons) may be envisioned as tiny loop currents, or magnetic dipoles.

Unpaired electrons may line-up the spin parallel to the external field, generating a net dipole moment **parallel** to the external field, or paramagnetism.

The orbiting motion of orbiting electrons may be modified by an external field (Lorentz force), generating a net dipole moment **anti-parallel** to the external field, or diamagnetism.

Example 2: estimate the net dipole moment of an orbiting electron in external field.

The net effect of magnetization is described by  $\vec{M}$ , defined as the the gross magnetic dipole moment per unit volume, in units of A/m. Again, in paramagnetic materials,  $\vec{M} \parallel \vec{B}_{ext}$ , and in diamagnetic materials,  $\vec{M} \parallel -\vec{B}_{ext}$ .

Magnetized materials generate additional magnetic field, which is just a dipole field by  $\vec{M}$  in a given volume, and is equivalent to the field generated by bound currents, defined as:

$$\vec{J}_b = \nabla \times \vec{M} \text{ (volume bound current), and}$$

$$\vec{K}_b = \vec{M} \times \hat{n} \text{ (surface bound current).}$$

You may envision  $\vec{J}_b$  and  $\vec{K}_b$  as the macroscopic effect of the net "atomic currents". Therefore, the magnetic field may be found using the familiar skills (e.g. Ampere's law), if the magnetization is given and bound currents determined.

Example 3: magnetic field by a uniformly magnetized long cylinder  $\vec{M} = M\hat{z}$ : only azimuthal bound currents are present. This is equivalent to a solenoid.

Example 4: magnetic field by a finite-length bar magnet (see additional notes).

Example 5: magnetic field by a uniformly magnetized sphere (see book): for this special geometry, the magnetic field inside the sphere is *uniform*, and outside the sphere is exactly a *dipole field*.

### II.3 Auxiliary field and linear media

The auxiliary field is defined as  $\vec{H} = \vec{B}/\mu_0 - \vec{M}$ . Therefore,  $\nabla \times \vec{H} = \vec{J}_f$ . Ampere's law may be used to find  $\vec{H}$  when distribution of the free current is symmetric.

For a linear medium,  $\vec{M} = \chi_m \vec{H}$ . So the total magnetic field is  $\vec{B} = \mu_0(\vec{H} + \vec{M}) = \mu_0(1 + \chi_m)\vec{H} = \mu\vec{H}$  inside the material. Outside the material,  $\vec{B} = \mu_0\vec{H}$ .

Example 6: find  $\vec{H}, \vec{B}, \vec{M}$  and bound currents inside and outside a solid cylinder with an axial free current.

Example 7: find  $\vec{H}, \vec{B}, \vec{M}$  and bound currents inside and outside a magnetized solenoid, if the solenoid is a solid cylinder or cylindrical shell.

Boundary conditions:  $\vec{B}_{out} - \vec{B}_{in} = \mu_0 \vec{K} \times \hat{n}$ . Note that  $\vec{K} = \vec{K}_f + \vec{K}_b$ .

Example 8: find  $\vec{B}$  inside and outside a magnetic sphere ( $\mu$ ) placed in a uniform field  $\vec{B}_0 = B_0 \hat{z}$ .