

Key to Solutions: Homework Problem Set 1

R2.2

There is no constraint in the particles' position, so the two particles can be anywhere with their positions x_1 and x_2 as $0 < x_1 < L$ and $0 < x_2 < L$.

The total kinetic energy of the two particles is given as $E < E_{tot} = E_1 + E_2 < E + \delta E$, where $E_1 = \frac{p_1^2}{2m}$ and $E_2 = \frac{p_2^2}{2m}$, and p_1 and p_2 are the momentum of the two particles, respectively. In the $p_1 - p_2$ phase space, the system occupies the states in an area of a circular shell with the inner radius $R = \sqrt{2mE}$ and the outer radius $R' = \sqrt{2m(E + \delta E)}$.

S1-15:

The number of micro-states is given by $\psi = \frac{VV_p}{h^3}$. Here, V is the volume of the sphere in the real space, V_p the volume of the momentum sphere, and h the Planck constant. So $V = \frac{4}{3}\pi r^3$ and $V_p = \frac{4}{3}\pi p_0^3$. With all the numbers, it is found that $\psi = 13$. So the proton has 13 microscopic states in the nucleus.

S1-21a:

The number of micro-states for this particle is given by $\psi = \frac{VV_p}{h^3}$. Here, h is the Planck constant, V is the volume of the room $V = 6 \times 8 \times 3 = 144 \text{ m}^3$, and V_p is the volume of the momentum sphere whose momentum radius is $R_p = \sqrt{2mE}$, or $V_p = \frac{4}{3}\pi R_p^3 = \frac{4}{3}\pi(2mE)^{\frac{3}{2}}$. Therefore, $\psi = \frac{\frac{4}{3}\pi(2m)^{\frac{3}{2}}VE^{\frac{3}{2}}}{h^3}$. The density of the states is $g(E) = \frac{\partial\psi}{\partial E} = \frac{2\pi(2m)^{\frac{3}{2}}VE^{\frac{1}{2}}}{h^3}$. Given all the numbers, $g(E) \approx 7.6 \times 10^{54} \text{ J}^{-1}$, or $g(E) \approx 1.2 \times 10^{36} \text{ eV}^{-1}$.