

Solutions to Homework Problem Set 10

Reif 5.7

(c) Using the results from (a) and (b) and by removing dp term, we get the relation between dT and dz as:

$$\frac{dT}{dz} = -\frac{\gamma - 1}{\gamma} \frac{\mu g}{R}$$

Here μ is the molar mass of N_2 , and $\mu = 28g$. We may then find $\frac{dT}{dz} = -9.4 \times 10^{-3} \text{ K/m} = -9.4 \text{ K/km}$. So the temperature decreases with the altitude and by about 10 K per kilometer.

(e) If the atmosphere is adiabatic, the temperature varies with the altitude as in (c)

$$\frac{dT}{dz} = -\frac{\gamma - 1}{\gamma} \frac{\mu g}{R}$$

which leads to

$$T = -\frac{\gamma - 1}{\gamma} \frac{\mu g}{R} z + T_0$$

On the other hand, we may integrate the adiabatic relation in (b) and get

$$P = P_0 \left(\frac{T}{T_0} \right)^{\frac{\gamma}{\gamma-1}}$$

Replace T with the previous expression, we get

$$P = P_0 \left(1 - \frac{\gamma - 1}{\gamma} \frac{\mu g z}{RT_0} \right)^{\frac{\gamma}{\gamma-1}}$$

Stowe 12.23 (a-c)

We will calculate the work W , heat Q , and change in internal energy ΔE for each other cases. For the isobaric expansion, P is a constant, so $W = PdV = P\Delta V = 1.01 \times 10^5 \text{ Pa} \times 5 \times 10^{-3} \text{ m}^3 = 5.06 \times 10^2 \text{ J}$. Using the ideal gas law, we also find the final temperature by $T_f = T_i \frac{V_f}{V_i} = 273 \text{ K} \times \frac{15}{10} = 410 \text{ K}$. So the change of the internal energy is $\Delta E = \frac{1}{2} \nu N k \Delta T = \frac{1}{2} \nu n R \Delta T = 1.27 \times 10^3 \text{ J}$. Using the first law, the heat added to the gas is given by $Q = W + \Delta E = 1.78 \times 10^3 \text{ J}$.

For the isothermal expansion, T is a constant, so $\Delta E \propto \Delta T = 0$. In an isothermal expansion, using the ideal gas law, we get the relation between P and V as $P = \frac{nRT}{V}$, so the work done by pressure is $W = \int_i^f PdV = nRT \int_i^f \frac{dV}{V} = nRT \ln\left(\frac{V_f}{V_i}\right) = 410 \text{ J}$. The heat added to the gas is $Q = W + \Delta E = W = 410 \text{ J}$.

For the adiabatic expansion, there is no heat exchange, so $Q = 0$. We use the adiabatic relation $PV^\gamma = P_i V_i^\gamma = P_f V_f^\gamma = k$, so the work done by pressure is

$$W = \int_i^f PdV = \int_i^f kV^{-\gamma} dV = \frac{k}{-\gamma + 1} (V_f^{-\gamma+1} - V_i^{-\gamma+1}) = \frac{P_i V_i - P_f V_f}{\gamma - 1}$$

. Since the gas molecule has 5 degrees of freedom, so $c_v = \frac{5}{2}R$ and $\gamma = (c_v + R)/c_v = 1.4$. Using the adiabatic relation, $P_f = P_i(\frac{V_i}{V_f})^\gamma = 0.57$ atm, and $W = \frac{P_i V_i - P_f V_f}{\gamma - 1} = 367$ J. From the first law $Q = W + \Delta E = 0$, so $\Delta E = -W = -367$ J. Or, in this problem, you may use the adiabatic relation $TV^{\gamma-1} = \text{const.}$ to find T_f and then ΔE , and $W = -\Delta E$.

Stowe 12.32

(a) We may derive the differential adiabatic relation using the second law and ideal gas law. By differentiating the ideal gas law, we get $PdV + VdP = nRdT$, and the second law is $dS = C_V dT + PdV = 0$ for the adiabatic process. Now we remove dT term from the two equations, we get $(\frac{nR+C_V}{C_V}PdV + VdP = 0$. Since $C_V + nR = C_P$ and $\frac{C_P}{C_V} = \gamma$, we get $\gamma PdV + VdP = 0$. So $B = -V\frac{\partial P}{\partial V}|_s = \gamma P$.

(b) the sound speed $v^2 = B/\rho = \gamma P/\rho$, so $v = \sqrt{\gamma P/\rho} = 341$ m/s.

(c) From the ideal gas law, $P = \frac{n}{V}RT = \frac{\rho}{\mu}RT$, where μ is the molar mass of the air. So $P/\rho = RT/\mu$, and $v = \sqrt{\gamma RT/\mu}$.