

Solutions to Homework Problem Set 16

1. Find the efficiency of the Carnot cycle in terms of T_h and T_l .

From the class notes, the total work done throughout the Carnot cycle is $W = RT_h \ln \frac{V_2}{V_1} + RT_l \ln \frac{V_4}{V_3}$, and heat is absorbed only during the isothermal expansion stage as $Q_h = RT_h \ln \frac{V_2}{V_1}$. The efficiency is defined as $\eta = \frac{W}{Q_h}$. Also noting that the 2nd and 4th stages are adiabatic, we may relate V_2 and V_3 and V_1 and V_4 using the adiabatic relation $TV^{\gamma-1} = \text{const.}$. For the 2nd stage of adiabatic expansion, $T_2 V_2^{\gamma-1} = T_3 V_3^{\gamma-1}$, thus $V_3 = V_2 (T_2/T_3)^{1/(\gamma-1)} = V_2 ((T_h/T_l)^{1/(\gamma-1)})$. In the same way, for the last stage of adiabatic compression, $T_4 V_4^{\gamma-1} = T_1 V_1^{\gamma-1}$, so $V_4 = V_1 (T_1/T_4)^{1/(\gamma-1)} = V_1 (T_h/T_l)^{1/(\gamma-1)}$. Replacing V_3 and V_4 in the expressions of η , we get $\eta = (RT_h \ln \frac{V_2}{V_1} + RT_l \ln \frac{V_4}{V_3}) / (RT_h \ln \frac{V_2}{V_1}) = (RT_h \ln \frac{V_2}{V_1} - RT_l \ln \frac{V_2}{V_1}) / (RT_h \ln \frac{V_2}{V_1}) = (T_h - T_l) / T_h$.

2. Find the efficiency of engine in Stowe 13.7 in terms of V_1 and V_3 .

(a) From the work and heat analysis in Stowe 13.7, we get the total work $W_{tot} = W_I + W_{II} + W_{III} + W_{IV} = 0 + P_2 V_1 [1 - (\frac{V_1}{V_3})^{\gamma-1}] / (\gamma-1) + 0 + P_1 V_1 [(\frac{V_1}{V_3})^{\gamma-1} - 1] / (\gamma-1) = (P_2 - P_1) V_1 [1 - (\frac{V_1}{V_3})^{\gamma-1}] / (\gamma-1)$. And the amount of absorbed heat is $Q_{abs} = Q_I = \frac{c_V}{R} (P_2 - P_1) V_1$. For the ideal gas, there is also the relation $c_V/R = c_V/(c_P - c_V) = 1/(\gamma - 1)$. The engine efficiency is therefore $\eta = W_{tot}/Q_{abs} = 1 - (\frac{V_1}{V_3})^{\gamma-1}$.

(b) The above expression of η may be written in terms of temperatures at the end of the four stages by using the adiabatic relation $TV^{\gamma-1} = \text{const.}$ to replace V_1/V_3 . In the engine cycle, it is seen that $V_1 = V_2$ (because of the first stage of isochoric heating). And in the second stage of adiabatic expansion, there is $T_2 V_2^{\gamma-1} = T_3 V_3^{\gamma-1}$. Therefore, $(\frac{V_1}{V_3})^{\gamma-1} = (\frac{V_2}{V_3})^{\gamma-1} = T_3/T_2$, and the engine efficiency is $\eta = 1 - (\frac{V_1}{V_3})^{\gamma-1} = 1 - T_3/T_2$.

Note that the above result can be also written as $\eta = 1 - (\frac{V_1}{V_4})^{\gamma-1} = 1 - T_4/T_1$.

3. Reif 5.22 (a) The heat pump works in a refrigeration cycle by extracting Q_o amount of heat from the outside at the temperature T_o and releasing Q_i amount of heat to the interior at the temperature T_i , where $T_i > T_o$. To abide by the second law, an amount of work W is needed so that $Q_i = Q_o + W$. Note that Q_i, Q_o, W given here are all positive. This way, the heat pump goes through a cycle with zero entropy change, and the net entropy change includes the entropy changes of the outside losing heat Q_o , thus $\Delta S_o = -\frac{Q_o}{T_o}$ and the interior absorbing heat Q_i , thus $\Delta S_i = \frac{Q_i}{T_i}$. Note that the interior and outside are both treated as heat reservoirs that do not change their temperatures by losing or absorbing the amount of heat discussed here. According to the second law, $\Delta S \geq 0$, so

$$-\frac{Q_o}{T_o} + \frac{Q_i}{T_i} = -\frac{Q_i - W}{T_o} + \frac{Q_i}{T_i} \geq 0$$

With $T_i > T_o$, the above relation can be reorganized as

$$\frac{Q_i}{W} \leq \frac{T_i}{T_i - T_o}$$

Therefore, the maximum amount of heat to be supplied to the interior is $W \frac{T_i}{T_i - T_o}$. For every kilowatt-hour work W , $\frac{T_i}{T_i - T_o}$ kilo-watt-hours heat can be supplied at the maximum.

(b) $\frac{T_i}{T_i - T_o} = \frac{273.15 + 25}{25} = 11.9$, so at the maximum, 11.9 kilowatt-hours heat is supplied for every kilowatt-hour work.