

Solutions to Homework Problem Set 2

Stowe 6.2

(a) For each particle, 2 "states" are possible, either in the front or the back of the room. Then for N particles, the total number of states is simply $\psi = 2^N = 2^{10^{28}}$. The form y^N may be written in the form of the powers of ten by $y^N = 10^{N \log(y)}$. Therefore, $\psi = 10^{10^{28} \log(2)} = 10^{3.0 \times 10^{27}}$.

(b) If we divide the room into quarters, each particle would have 4 "states", i.e., in any of the four quarters of the room. Then for N particles, the total number of states is $\psi = 4^N = 4^{10^{28}} = 10^{10^{28} \log(4)} = 10^{6.0 \times 10^{27}}$.

Stowe 6.7

(a) The total number of quantum states for 1 particle in a 3D space is given by $\psi = \frac{VV_p}{h^3}$. Here V is the volume of the box $V = 1 \text{ cm} \times 1 \text{ cm} \times 2 \text{ cm} = 2 \times 10^{-6} \text{ m}^3$. V_p is the volume of the momentum sphere with the radius $R_p = p_{max} = 3 \times 10^{-24} \text{ kg m/s}$. So V_p can be calculated by $V_p = \frac{4}{3}\pi R_p^3 = 1.1 \times 10^{-70} (\text{kg m/s})^3$. h is Planck's constant. Therefore, we arrive at $\psi = 7.8 \times 10^{23}$. The probability for the particle in any of these states is therefore $P = \frac{1}{\psi} = 1.3 \times 10^{-24}$.

(b) The number of states for the particle to have the magnitude of the momentum smaller than $p_1 = 2 \times 10^{-24} \text{ kg m/s}$ can be calculated to be $\psi_1 = \frac{VV_{p_1}}{h^3}$. Then the probability of this particle with the magnitude of the momentum smaller than p_1 is just the number of states ψ_1 divided by the total number of states ψ , or $P = \frac{\psi_1}{\psi} = 0.30$, or 30% probability.

(c) We first consider the number of states for the particle in the box with the magnitude of momentum smaller than $p' = 10^{-24} \text{ kg m/s}$, which is $\psi' = \frac{VV_{p'}}{h^3} = 2.9 \times 10^{22}$. Then the number of states of the particle in the box with the magnitude of the momentum between p' and p_{max} is $\psi_2 = \psi - \psi' = 7.5 \times 10^{23}$. Further, the number of states for this particle in only half of the box would be halved as well, as $V \rightarrow \frac{1}{2}V$ in the calculation of the number of the states. Therefore, the number of states in this problem becomes $\frac{1}{2}\psi_2$, and the probability is the number of states in this situation divided by the total number of the states: $P = \frac{\frac{1}{2}\psi_2}{\psi} = 0.48$.

Stowe 6-20:

(a) Let Ω be the number of states in a certain energy range between E and $E + \delta E$, then we have $\Omega(E, \delta E) \propto E^{\frac{f}{2}} \delta E$, where f is the number of the degrees of freedom in the system. When E is doubled, $\Omega(2E, \delta E) \propto (2E)^{\frac{f}{2}} \delta E$. So the number of states increases by a factor of $\frac{\Omega(2E, \delta E)}{\Omega(E, \delta E)} = 2^{\frac{f}{2}} = 2^{2 \times 10^{24}} = 10^{2 \times 10^{24} \log(2)} = 10^{6.0 \times 10^{23}}$.

(b) Same as in (a), when E increases to $1.01E$, the number of states increases by a factor of $\frac{\Omega(1.01E, \delta E)}{\Omega(E, \delta E)} = 1.01^{\frac{f}{2}} = 1.01^{2 \times 10^{24}} = 10^{2 \times 10^{24} \log(1.01)} = 10^{8.6 \times 10^{21}}$. It is seen that a tiny increase in the internal energy can SIGNIFICANTLY increase the number of states of the system.