

Solutions to Homework Problem Set 21

Reif 6.3

As the magnetic energy is calculated by $\epsilon = -\vec{\mu} \cdot \vec{H}$, the parallel spin has a lower energy $\epsilon_+ = -\mu H$. The probability of parallel polarization is

$$P_+ = \frac{e^{-\epsilon_+/kT}}{e^{-\epsilon_+/kT} + e^{-\epsilon_-/kT}} = \frac{e^{\mu H/kT}}{e^{\mu H/kT} + e^{-\mu H/kT}}$$

If $P_+ = 0.75$, with some algebra, there is

$$T_c = \frac{2\mu H}{k \ln 3}$$

(a) For Bohr magneton, the above expression yields $T_c = 3.7$ K.

(b) For protons, the above expression yields $T_c = 5.6 \times 10^{-3}$ K.

Be careful with converting the cgs units to metric units: $1 \text{ Tesla} = 10^4 \text{ Gauss}$, $1 \text{ Joule} = 10^7 \text{ erg}$.

Stowe 17.17

(a) Using the expression for Curie constant,

$$C = (\sum_{l_z, s_z} 1)^{-1} (\sum_{l_z, s_z} (l_z + 2s_z)^2) \left(\frac{N_a \mu_B^2}{k} \right)$$

where μ_B is the Bohr magneton

$$\mu_B = \frac{e\hbar}{2m_e} = 9.27 \times 10^{-24} \text{ J/T}$$

The sum is over all possible l_z and s_z combinations. In this problem, there are altogether 4 possible combinations as given, thus $\sum_{l_z, s_z} 1 = 4$, and $\sum_{l_z, s_z} (l_z + 2s_z)^2 = (1 - 1)^2 + (0 + 1)^2 + (0 - 1)^2 + (-1 + 1)^2 = 2$. Therefore, $C = \frac{2}{4} \frac{N_a \mu_B^2}{k} = 1.88 \text{ JK/T}^2$.

(b) At $B = 2\text{T}$, $T = 10^3\text{K}$, $\mu_B B/kT \approx 0.001 \ll 1$, so we can find the magnetic moment using the approximation expression at high temperature limit $M = N_a \bar{\mu} = C \frac{B}{T} = 3.76 \times 10^{-3} \text{ J/T}$.

At $T = 10$ K, $\mu_B B/kT \approx 0.1$, reasonably smaller than 1, the approximation to high temperature limit yields $M = N_a \bar{\mu} = C \frac{B}{T} = 0.38 \text{ J/T}$. Calculation using the exact expression for $\bar{\mu}$ (see class notes) gives

$$\bar{\mu} = \mu_B \frac{e^{\mu_B B/kT} - e^{-\mu_B B/kT}}{2 + e^{\mu_B B/kT} + e^{-\mu_B B/kT}} = 0.07 \mu_B$$

Thus $M = N_a \bar{\mu} \approx 0.39 \text{ J/T}$, very close to the approximation calculation.

(c) As the transition between the low and high temperature regimes is at when $\mu_B B \approx kT$, we get the transition temperature at which the solid is nearly completely magnetized as $T_c = \mu_B B/k =$

6.7×10^{-5} K. In fact, this transition temperature is a very crude estimate. If we use the exact expression in (b), it is seen that at this estimated T_c , $\bar{\mu} \approx 0.5\mu_B$, about 50% magnetization.

We may derive a more accurate result using the expression for $\bar{\mu}$ in (b), which can be further written as

$$\bar{\mu} = \mu_B \frac{e^\eta - e^{-\eta}}{2 + e^\eta + e^{-\eta}} = \mu_B \frac{(e^{1/2\eta} + e^{-1/2\eta})(e^{1/2\eta} - e^{-1/2\eta})}{(e^{1/2\eta} + e^{-1/2\eta})^2} = \mu_0 \frac{e^{1/2\eta} - e^{-1/2\eta}}{e^{1/2\eta} + e^{-1/2\eta}} = \mu_B \frac{e^\eta - 1}{e^\eta + 1}$$

where $\eta = \frac{\mu_B B}{kT}$. When $\eta \gg 1$, the maximum magnetization is $\bar{\mu} \approx \mu_B$, or the material is completely magnetized. Suppose that when $\bar{\mu} = 0.99\mu_B$, the material is considered to be completely magnetized, then there is

$$\frac{e^\eta - 1}{e^\eta + 1} = 0.99$$

yielding $e^\eta = 199$, or

$$T_c = \frac{\mu_B B}{k \ln 199} = 1. \times 10^{-5} \text{K}$$