

Solutions to Homework Problem Set 22

Stowe 19.9

From the definition, the occupation number is give by

$$\bar{n} = \frac{\sum_n n e^{-n(\epsilon-\mu)\beta}}{\sum_n e^{-n(\epsilon-\mu)\beta}}$$

The sum is over all allowed values of n . In this problem, n can only take the values of 0, 1, and 2. So we may work out the sums to find

$$\bar{n} = \frac{e^{-(\epsilon-\mu)\beta} + 2e^{-2(\epsilon-\mu)\beta}}{1 + e^{-(\epsilon-\mu)\beta} + e^{-2(\epsilon-\mu)\beta}}$$

Stowe 19.16

In this problem, we first can find $\eta = \beta(\epsilon - \mu)$ where $\beta = 1/(kT)$. For $T = 1000$ K, $\eta = 0.1\text{eV} \times 1.6 \times 10^{-19}\text{J/eV}/(1.38 \times 10^{-23}\text{J/K} \times 1000\text{K}) = 1.16$.

- (a) For classical statistics, $\bar{n} = e^{-\eta} = 0.31$
- (b) For BE statistics, $\bar{n} = (e^{-\eta} - 1)^{-1} = 0.46$
- (c) For FD statistics, $\bar{n} = (e^{-\eta} + 1)^{-1} = 0.24$

It's seen that the occupation number for BE statistics is the largest, followed by the MB statistics and then by FD statistcis.

(d) at $T = 300$ K, $\eta = 3.86$, and the occupation number of MB, BE, and FD statistics is 0.02, 0.02, and 0.02, respectively, all nearly the same. The reason is that at lower temperature, particles tend to occupy lower energy states, so the probability of particles occupying this particular energy state is very low for all particles.

Note: from the above exercise, it appears as if at lower temperatures, the FD and BE distributions are reduced to classical MB distribution. However, this is misleading! In the problem, the underlying assumption is that the $\epsilon - \mu$ does not depend on the temperature, which, however, is a poor assumption. The chemical potential of particles is a function of temperature. Physically, at high temperature, the number of quantum states accessible to the system rapidly increases so that the occupation number of all three kinds of particles should become very similar, i.e., $\bar{n} \ll 1$ in all three cases. However, at very low temperature, the distinction in \bar{n} for three particles is sharp. At low temperatures, particles tend to occupy the lower energy states (think about the Boltzmann factor). Bosons tend to swarm into the lowest energy state (the Bose-Einstein Condensation), so that $\bar{n} \approx N$ at the lowest energy state (the ground state) but $\bar{n} \approx 0$ in any other excited state. For Fermions, due to exclusion principle, $\bar{n} \approx 1$ for states with energy lower than a certain energy determined by the number of particles, and in states with larger energy, $\bar{n} \approx 0$. The behavior of \bar{n}

for classical particles is in between, although, at very low temperature, the condition for classical approximation is no longer valid.

Stowe 19.23

(a) For distinguishable classical particles, there are 8 different ways for 3 particles to occupy 2 quantum states, and 2 different ways that all three particles are in the same state. So the probability of all three particles being in the same state is $2/8 = 1/4$.

(b) If particles are not distinguishable, then there are only 4 different ways to occupy the 2 states. The probability of all three particles occupying the same state is $2/4 = 1/2$.

(c) For Bosons, the situation is the same as undistinguishable classical particles, so the probability is $2/4 = 1/2$.

(d) According to exclusion principle, no two particles may occupy the same state, so the probability is 0.