

### Solutions to Homework Problem Set 23

#### Stowe 19.26

(a) According to the density of copper, it is seen that every  $1\text{cm}^3$  of copper contains  $N$  copper atoms, and  $N$  is given by  $N = \frac{8.9\text{g/cm}^3}{64 \times 1.67 \times 10^{-24}\text{g/atom}} = 8.3 \times 10^{22}$ . So the volume per conduction electron is  $v = V/N = 1.2 \times 10^{-29}\text{m}^3$ .

(b) The characteristic separation between any two conduction electrons is therefore  $r = (V/N)^{1/3} = 2.3 \times 10^{-10}\text{m}$ , or  $r = 0.23\text{nm}$ .

(c) At room temperature,  $T \approx 300\text{K}$ , the mean momentum of the electron is  $p \approx \sqrt{3mkT} = \sqrt{3 \times 0.91 \times 10^{-30}\text{kg} \times 1.38 \times 10^{-23}\text{J/K} \times 300\text{K}} = 1.1 \times 10^{-25}\text{kgm/s}$ . We then estimate  $rp/h = 2.3 \times 10^{-10}\text{m} \times 1.1 \times 10^{-25}\text{kgm/s} / 6.63 \times 10^{-34}\text{Js} \approx 0.04 \ll 1$ . So the conduction electrons cannot be treated using classical statistics.

(d) At  $T = 10,000\text{K}$ , the mean momentum of the electron will be larger by a factor of  $\sqrt{10000/300} \approx 6$ , so the  $rp/h \approx 0.2$ . This is not too much larger than 1, so at this high temperature, conduction electrons may be treated by classical statistics but it is not a very good approximation.

#### Stowe 21.2

(a) Starting from the Planck's function, at the long wavelength limit, the exponent  $\beta hc/\lambda \ll 1$ , and we may expand the exponential function so that  $du/d\lambda \approx 8\pi hc \frac{\lambda^{-5}}{1 + \beta hc/\lambda - 1} = 8\pi hc \frac{\lambda^{-4}}{\beta hc} = 8\pi kT \lambda^{-4}$ . The  $\lambda^{-4}$  dependence of long-wavelength radiation was found as the Rayleigh-Jeans law. Note that in the Rayleigh-Jeans law, the Planck's constant is NOT present!

(b) For short wavelength, the exponential term is much larger than 1, so  $du/d\lambda \approx 8\pi hc \frac{\lambda^{-5} e^{-\beta hc/\lambda}}{e}$ .

#### Stowe 21.12

(a) We will use the Stefan-Boltzmann's law  $J = \sigma T^4$ , where  $J$  is the radiation flux and  $\sigma = 5.67 \times 10^{-8}\text{W/m}^2/\text{K}^4$ . This relation is derived by integrating Planck's function of radiation intensity (flux per unit wavelength or frequency) over all wavelengths (or frequencies). The total radiation energy per second over a radiating surface of a stellar sphere is therefore  $W = 4\pi R^2 J$  J/s, where  $R$  is the radius of the star. For the Sun,  $T = 5800\text{K}$ ,  $R = 7 \times 10^8\text{m}$ , so  $W = 5.67 \times 10^{-8}\text{W/m}^2/\text{K}^4 \times (5800\text{K})^4 \times 4\pi \times (7 \times 10^8\text{m})^2 = 4.0 \times 10^{26}\text{J/s}$ .

(b) From the energy density  $u = aT^4$  where  $a = 7.56 \times 10^{-16}\text{J/m}^3/\text{K}^4$ , we may derive the total energy stored inside the sun as  $U = uV = 4/3\pi R^3 aT^4 = 1.1 \times 10^{40}\text{J}$ .

(c) The "lifetime" would be  $\tau = U/W = 2.8 \times 10^{13}\text{s} \approx 8.7 \times 10^5\text{yr}$ . So the sun may radiate at the current rate for about a million year if the nuclear fusion stops.