

## Solutions to Homework Problem Set 25

### Stowe 20.4

In the phase space, the number of states in a given energy range  $\epsilon$  and  $\epsilon+d\epsilon$  is given by  $dN_Q = g(\epsilon)d\epsilon$ , where  $g(\epsilon)$  is the density of states (per unit energy). For massive nonrelativistic particles,  $g(\epsilon) = \frac{2\pi V}{h^3}(2m)^{3/2}\epsilon^{1/2}J^{-1}$  if no spin is considered. The energy at  $\epsilon = kT = 1.38 \times 10^{-23}J/K \times 295K = 4.07 \times 10^{-21}J$ .

(a) Using the expression above, it follows that  $g(\epsilon) = \frac{2\pi 30m^3}{(6.63 \times 10^{-34}Js)^3}(2 \times 4.8 \times 10^{-26}kg)^{3/2} \times (4.07 \times 10^{-21}J)^{1/2}J^{-1} \times 1.6 \times 10^{-19}J/eV = 1.96 \times 10^{35}eV^{-1}$ . So there are nearly  $10^{35}$  states per 1 eV.

(b) Electrons are Fermions. Each electron has 2 spin states of the same energy, so  $g(\epsilon) = 2 \times \frac{2\pi V}{h^3}(2m)^{3/2}\epsilon^{1/2}J^{-1} = \frac{4\pi 10^{-5}m^3}{(6.63 \times 10^{-34}Js)^3}(2 \times 0.91 \times 10^{-30}kg)^{3/2} \times (4.07 \times 10^{-21}J)^{1/2}J^{-1} \times 1.6 \times 10^{-19}J/eV = 1.08 \times 10^{22}eV^{-1}$ . So there are nearly  $10^{22}$  states per 1 eV.

### Stowe 20.5

The particle number distribution is given by  $dN = n(\epsilon)dN_Q = n(\epsilon)g(\epsilon)d\epsilon$ .

(a) For bosons, we use the Bose-Einstein distribution so that

$$n(\epsilon) = \frac{1}{e^{(\epsilon-\mu)/kT} - 1} = \frac{1}{e^{1+0.01eV \times 1.60 \times 10^{-19}J/eV / (1.38 \times 10^{-23}J/K \times 295K)} - 1} \approx 0.33$$

Using the result of  $g(\epsilon)$  from 20.4(a), we get  $dN/d\epsilon = 0.33 \times 1.96 \times 10^{35}eV^{-1} = 6.5 \times 10^{34}eV^{-1}$ .

(b) For electrons, we use the Fermi-Dirac distribution so that

$$n(\epsilon) = \frac{1}{e^{(\epsilon-\mu)/kT} + 1} = \frac{1}{e^{1-0.08eV \times 1.60 \times 10^{-19}J/eV / (1.38 \times 10^{-23}J/K \times 295K)} + 1} \approx 0.90$$

Using the result of  $g(\epsilon)$  from 20.4(b), we get  $dN/d\epsilon = 0.90 \times 1.08 \times 10^{22}eV^{-1} = 9.7 \times 10^{21}eV^{-1}$ .

### Stowe 22.26

In this problem, we compare the temperature  $T$  with the Fermi temperature  $T_F$  for two kinds of fermions, protons and electrons, inside the Sun. If  $T \ll T_F$ , then the fermions are nearly degenerate. To calculate the Fermi temperature, we first need to find the particle number density  $N/V$ . Since all the mass is contributed by protons (the electrons mass is smaller by 3 orders of magnitude hence can be ignored), so the number of protons (which is also the number of electrons) per  $m^3$  is  $N_p/V = N_e/V = \rho/m_p = 1.50 \times 10^5 kg/m^3 / 1.67 \times 10^{-27}kg = 8.98 \times 10^{31}m^{-3}$ . The Fermi temperature is calculated by  $T_F = \epsilon_F/k = (\frac{3}{8\pi} \frac{N}{V})^{2/3} \frac{h^2}{2mk}$ .

For protons, we use  $m = m_p = 1.67 \times 10^{-27}kg$ , so  $T_F \approx 4.6 \times 10^3K$ . Since  $T_F \ll T$ , so protons are not degenerate inside the Sun. For electrons, we use  $m = m_e = 0.91 \times 10^{-30}kg$ , so  $T_F \approx 8.5 \times 10^6K < T$ . So inside the Sun, electrons are not degenerate, either. In fact, inside the

Sun, the gravitational compression is supported by gas pressure (and radiation pressure), not by electron or proton degenerate pressure.