

Solutions to Homework Problem Set 3

Stowe 4-9:

(a) The rotation inertial is given by $I = 2mr^2$ where m is the mass of the Ni atom and r is the half separation between the two Ni atoms. With the numbers given, we may get $I = 2 \times 2.34 \times 10^{-26} \text{ kg} \times (0.55 \times 10^{-10})^2 \text{ m}^2 = 1.42 \times 10^{-46} \text{ kgm}^2$.

(b) The energy required to excite the molecule to the first excited rotational state is $\epsilon = \frac{1}{2I} L_1^2$ where L_1 is the angular momentum of the first excited rotational state $L_1 = 1(1+1)\hbar^2 = 2\hbar^2$. So $\epsilon = 7.84 \times 10^{-23} \text{ J} = 4.90 \times 10^{-4} \text{ eV}$.

(c) The minimum temperature needed to excite the first rotational state may be estimated by comparing the energy from (b) with the thermal energy kT . With $\epsilon = kT$, we arrive at $T = 5.7 \text{ K}$ degree.

So the molecules can be easily excited to the rotating states.

Stowe 6-22

(a) According to the equipartition theorem, the thermal energy of the system is $E_{th} = \frac{1}{2}kTf$ where $\frac{1}{2}kT$ is the mean energy per degree of freedom, and $f = \nu N$ is the number of degrees of freedom of the system. With the numbers given, $E_{th} = 3.8 \times 10^4 \text{ J}$.

(b) The translational motion of a particle has 3 degrees of freedom, and the mean energy in each degree of the freedom is $\frac{1}{2}kT$. Therefore, the mean kinetic energy of a particle is $\epsilon_{tra} = \frac{1}{2m}(p_x^2 + p_y^2 + p_z^2) = \frac{3}{2}kT$, and the root-mean-square momentum of the particle is given by $p_{rms} = \sqrt{p_x^2 + p_y^2 + p_z^2} = \sqrt{3mkT} = 3.2 \times 10^{-23} \text{ kg m/s}$. Suppose that the maximum momentum is twice p_{rms} , then $p_0 = 6.4 \times 10^{-23} \text{ kg m/s}$. And the volume of the momentum sphere is therefore $V_p = \frac{4}{3}\pi p_0^3 = 1.1 \times 10^{-66} \text{ (kgm/s)}^3$.

(c) The total number of quantum states for the translational motion is $\psi = \frac{VV_p}{h^3} = 5.7 \times 10^{32}$.