

### Solutions to Homework Problem Set 4

Reif 2-10:

The work done by pressure is (if  $\gamma \neq 1$ )

$$W = \int_{V_i}^{V_f} P dV = \int_{V_i}^{V_f} K V^{-\gamma} dV = \frac{K}{1-\gamma} (V_f^{1-\gamma} - V_i^{1-\gamma})$$

Using  $K = PV^\gamma = P_i V_i^\gamma = P_f V_f^\gamma$ , the above result can be further deduced to

$$W = \frac{P_f V_f - P_i V_i}{1-\gamma}$$

Reif 2-11:

When the system evolves quasi-statically and adiabatically (i.e., in thermal insulation) from macrostate A to B, the change in the internal energy is given according to the First Law:

$$\Delta E = E_B - E_A = -W = - \int_{V_A}^{V_B} P dV$$

Using the result from R.2-10,

$$W = \frac{P_B V_B - P_A V_A}{1-\gamma} = \frac{(8 \times 10^9 - 32 \times 10^9) \text{erg}}{1-5/3} = 36 \times 10^9 \text{erg} = 3600\text{J}$$

Therefore,  $\Delta E = E_B - E_A = -W = -3600\text{J}$ .

(a) Along route a, the system first expands at the constant pressure  $P = P_A$  from  $V_A$  to  $V_B$  and does work, then the pressure reduces to  $P_B$  but without any change in the volume hence not doing any work. So the amount of work by the system is  $W = \int P dV = P_A(V_B - V_A) = 32 \times (8 - 1) \times 100\text{J} = 22400\text{J}$ .

Using the first law, the amount of heat is therefore  $Q = W + \Delta E = (22400 - 3600)\text{J} = 18800\text{J}$ .  $Q$  is positive, indicating that the system absorbs heat.

(b) Along route b, the linear relation between  $P$  and  $V$  is

$$\frac{P - P_A}{P_B - P_A} = \frac{V - V_A}{V_B - V_A}$$

. With this relation, the integral  $W = \int_{V_A}^{V_B} P dV = 11550\text{J}$ . Or in a simpler way, the work done is the area in the P-V diagram under the b curve, therefore,  $W = 1/2(V_A + V_B)(P_A - P_B) = 11550\text{J}$ .  $W$  is positive because the system is expanding to larger  $V$  with decreasing  $P$ , doing a positive work.

Using the first law,  $Q = \Delta E + W = 7950\text{J}$ .  $Q$  is positive indicating that the system absorbs heat.

(c) The work integral can be done in two steps, A-A', and A'-B. No work is done from A-A' since the volume does not change. So  $W = P_B(V_B - V_A) = 700\text{J}$ , and  $Q = \Delta E + W = -2900\text{J}$ . Along this route, the system releases heat.