

Solutions to Homework Problem Set 5

Stowe 5.23:

(a) Since the gas is expanding, the work is done by the gas, and the amount of work in this simple case is $W = P\Delta V = 150 \text{ atm} \times 1.013 \times 10^5 \text{ Pa/atm} \times 0.01 \text{ m}^3 = 1.52 \times 10^5 \text{ J}$.

(b) Since no heat transfer is allowed, according to the first law, $\Delta E = Q - W = -W$. On the other hand, according to the equipartition theorem, the internal energy of the gas is given by $E = \frac{1}{2}fkT$, so $\Delta E = \frac{1}{2}fk\Delta T$. Therefore, $\Delta T = \frac{2\Delta E}{fk} = \frac{-2W}{fk} = -22 \text{ K}$. So the internal energy of the system decreases to do the work, and the temperature of the gas decreases by 22 K.

Stowe 5.25 (c-e)

Given a differential $dF = g(x, y)dx + h(x, y)dy$, by testing whether the relation

$$\frac{\partial g(x, y)}{\partial y} = \frac{\partial h(x, y)}{\partial x}$$

holds, we can tell that differentials in (d, e) are exact differentials, and the differential in (c) is an inexact differential.

By integration by parts, we can also find a function $F(x, y)$ in (d, e) cases:

$$(d) F(x, y) = xe^xy + \text{const.}$$

$$(e) F(x, y) = x^4y^{-2} + \text{const.}$$

Such a function cannot be found for (c).

Stowe 7.16

From the multiplicative property of the number of states, there is $\Omega_0 = \Omega_1(E_1)\Omega_2(E_2)$. When there is a small change in energy $\delta \ll E_1, \delta \ll E_2$, we may use Taylor series expansion to estimate the change in Ω_0 . For convenience, we take the logarithm of Ω_0 as $\ln(\Omega_0) = m_1\ln E_1 + m_2\ln E_2$, where $m_1 = 10^{24}$, and $m_2 = 2 \times 10^{24}$. When the energy changes by $\delta = 0.01$, we write the total number of states as Ω'_0 , so

$$\ln(\Omega'_0) = m_1\ln(E_1 + \delta) + m_2\ln(E_2 - \delta) = \ln\Omega_0 + m_1\ln(1 + \delta/E_1) + m_2\ln(1 - \delta/E_2)$$

We apply Taylor expansion to the items with δ and keep terms up to the second order, then we get

$$\ln\Omega'_0 - \ln\Omega_0 = \left(\frac{m_1}{E_1} - \frac{m_2}{E_2}\right)\delta - \frac{1}{2}\left(\frac{m_1}{E_1^2} + \frac{m_2}{E_2^2}\right)\delta^2$$

. Since the first-order term is not zero (meaning that the two sub-systems are not in thermal equilibrium as the example we showed in the class), we may ignore the 2nd order term. Therefore,

$$\ln\Omega'_0 - \ln\Omega_0 = \ln\left(\frac{\Omega'_0}{\Omega_0}\right) = \left(\frac{m_1}{E_1} - \frac{m_2}{E_2}\right)\delta = -1.7 \times 10^{21}$$

And

$$\frac{\Omega'_0}{\Omega_0} = e^{-1.7 \times 10^{21}} = 10^{-1.7 \times 10^{21} \log_{10}(e)} = 10^{-7.4 \times 10^{20}}$$

It is shown that when the energy distribution changes by less than 1%, the number of states changes by nearly 10^{21} orders of magnitude. In this particular case, the change is further away from the thermal equilibrium - in thermal equilibrium, energy should be distributed in such a way that $\bar{E}_1 = \frac{1}{3}(E_1 + E_2)$, and $\bar{E}_2 = \frac{2}{3}(E_1 + E_2)$ (why?), and the number of states decreases sharply. The result indicates that the further away from the thermal equilibrium, the smaller (by many orders of magnitude!) is the probability of the system staying in such an energy distribution.