

Solutions to Homework Problem Set 8

Stowe 8.30

(a) The entropy change is derived using the second law $TdS = dE + dW$. In this case, we assume that the internal energy of the magnet does not change at a constant temperature, and the work is given by $dW = -Bd\mu$, so the change of entropy is given by $\Delta S = \int ds = -\frac{1}{T} \int Bd\mu$. Let's first find the relation between μ and B so as to proceed with the integral. We may assume that the magnet is a linear material so that $d\mu = CdB$, where C is a constant to be determined. In this problem, μ and B both increase by 1%, or $\frac{d\mu}{\mu} = \frac{dB}{B}$ ($= 1\%$). Or, $d\mu = \frac{\mu}{B}dB = \frac{\mu_0}{B_0}dB$, where μ_0 and B_0 are values in the initial state. The change of entropy can then be calculated as

$$\Delta S = \int ds = -\frac{1}{T} \frac{\mu_0}{B_0} \int BdB \approx -\frac{1}{T} \mu_0 \Delta B = -\frac{1}{300\text{K}} \times 10^{-3} \text{J/T} \times 10^{-3} \text{T} = -3.3 \times 10^{-9} \text{ J/k}$$

Note that in the above calculation, we retained only the first order term.

The number of accessible states changes by the factor $\Omega_f/\Omega_i = e^{\Delta S/k} = e^{-2.4 \times 10^{14}} = 10^{-1.0 \times 10^{14}}$. So the number of states decreases by 10^{14} orders of magnitude.

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By the second law, the change of entropy is given by $\Delta S = \frac{Q}{T}$, and the change of the number of states is $\Omega_f/\Omega_i = e^{\Delta S/k} = e^{Q/kT}$. Therefore,

$$T = \frac{Q}{k \ln(\Omega_f/\Omega_i)} = \frac{1\text{J}}{1.38 \times 10^{-23} \text{J/K} \times 10^{19} \times \ln 10} = 3.1 \times 10^3 \text{K}$$

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(a) Since the particles only have translational motion in 1 dimension, the number of degrees of freedom for one particle is $\nu = 1$, and the number of degrees of freedom for N particles is $f = \nu N = N$.

(b) The number of quantum states may be calculated by the number of "cells" in the phase diagram. For one particle, the number of quantum states is given by $\Omega = \frac{VV_p}{h}$, and in a one-dimensional box, $V = L$, $V_p = \sqrt{2mE}$, so the number of states for one particle is $\Omega_1 = BLE^{\frac{1}{2}}$, where B contains all the constants not related to L and E . For N particles, the number of states is $\Omega \propto \Omega_1 \Omega_2 \dots \Omega_N = BL^N E^{\frac{N}{2}}$, where B contains constants not related to L and E .

(c) The entropy of the system is given by $S = k \ln \Omega = kN \ln L + \frac{1}{2} kN \ln E + C$, where C is a constant.

(d) From the combined first and second law, we get $dE = dQ - dW = TdS - FdL$ in this case. And it is easily rewritten as $dS = \frac{1}{T} dE + \frac{F}{T} dL$.

(e) By definition, the generalized force, in this case, the pressure force F in one dimension, is given by $\frac{F}{T} = \left(\frac{\partial S}{\partial L}\right)_{E,N}$. On the other hand, from the expression of entropy in (c), we get $\left(\frac{\partial S}{\partial L}\right)_{E,N} = \frac{kN}{L}$. So we have $\frac{F}{T} = \frac{kN}{L}$, leading to the equation of state, or the ideal gas law for the 1-dimensional gas: $FL = NkT$.