

Physics 425, Spring 2009

Quiz 1 - Jan 22

Name: _____ ID: _____

(books and notes are not allowed)

1. Consider a system consisting of 4 spin particles. Each particle has a magnetic moment μ which can point either up or down. The total magnetic moment of the system is noted as M .
 - (a) How many spin configurations (i.e., number of microscopic states) can the system have? Suppose that the particles are distinguishable.
 - (b) How many different values (i.e., the number of macroscopic states) can M take?
 - (c) What is the probability of $M = 0$ in this system?

2. Consider a gas particle in a 2D box, i.e., $0 < x < L_x$, $0 < y < L_y$, where x and y are spatial coordinates of the particle, and L_x and L_y indicate the sizes of the box. The maximum kinetic energy of the particle is E .
 - (a) Using the uncertainty principle, i.e., $\Delta x \Delta p_x > h$, $\Delta y \Delta p_y > h$, where h is Planck constant, and p_x and p_y indicate the momentum of the particle, calculate the number of microscopic states $\psi(E)$ of this particle.
 - (b) If the maximum kinetic energy of the particle is increased to $2E$, is the number of states of the particle increased or decreased? By what factor?
 - (c) The kinetic energy of the particle is quantized. Find the expression for possible values of the kinetic energy of this particle (given that the maximum kinetic energy is E).

Solutions:

1. (a) total number of microstates: $\psi = 2^4 = 16$
 - (b) 5 possible values: -4μ , -2μ , 0 , 2μ , 4μ
 - (c) there are 6 possible combinations that correspond to $M = 0$, so the probability is $6/16 = 37.5\%$

2. (a) the total number of micro-states is the volume of the phase space, which is VV_p , divided by the volume of the smallest possible cell, which is the minimum of $\Delta x \Delta p_x \Delta y \Delta p_y = h^2$. Or, $\psi = VV_p/h^2$. In this 2D problem, $V = L_x L_y$, V_p is the volume of

the 2D momentum sphere with a radius of $\sqrt{2mE}$, $V_p = \pi(\sqrt{2mE})^2 = 2\pi mE$. Therefore,

$$\Psi = \frac{L_x L_y 2\pi mE}{h^2}.$$

(b) when the maximum kinetic energy is increased to $2E$, from the expression in (a), the number of states increases by a factor of 2.

(c) The momentum of the particle is quantized, so is the kinetic energy. Since the particle can be anywhere in the 2D box, $\Delta x = L_x$, $\Delta y = L_y$, and the smallest possible change of the momentum is $\Delta p_x = h/L_x$, $\Delta p_y = h/L_y$. The momentum of the particle can only take the following values: $p_x = n_x \Delta p_x$, and $p_y = n_y \Delta p_y$, where n_x and n_y are integers. The kinetic energy of the particle thus can take the following values:

$$\varepsilon = \frac{p_x^2 + p_y^2}{2m} = \frac{h^2}{2m} \left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} \right). \text{ [you are in a good shape if you reached this point!]}$$

Here, n_x and n_y can take the following values: $n_x = 0, 1, 2, \dots, N_x$, $n_y = 0, 1, 2, \dots, N_y$. N_x and N_y are the largest number n_x and n_y can take, which can be determined by the maximum kinetic energy of the particle: $N_x = \frac{\sqrt{2mE}}{\Delta p_x} = \frac{\sqrt{2mEL_x}}{h}$ (when the particle's momentum is only in the x-direction), and $N_y = \frac{\sqrt{2mE}}{\Delta p_y} = \frac{\sqrt{2mEL_y}}{h}$ (when the particle's momentum is only in the y-direction).

