Physics 425, Spring 2009

Quiz 1 - Jan 22

Name: ___________________      ID: _________________
(books and notes are not allowed)

1. Consider a system consisting of 4 spin particles. Each particle has a magnetic moment $\mu$ which can point either up or down. The total magnetic moment of the system is noted as $M$.
   (a) How many spin configurations (i.e., number of microscopic states) can the system have? Suppose that the particles are distinguishable.
   (b) How many different values (i.e., the number of macroscopic states) can $M$ take?
   (c) What is the probability of $M = 0$ in this system?

2. Consider a gas particle in a 2D box, i.e., $0 < x < L_x$, $0 < y < L_y$, where $x$ and $y$ are spatial coordinates of the particle, and $L_x$ and $L_y$ indicate the sizes of the box. The maximum kinetic energy of the particle is $E$.
   (a) Using the uncertainty principle, i.e., $\Delta x \Delta p_x > h$, $\Delta y \Delta p_y > h$, where $h$ is Planck constant, and $p_x$ and $p_y$ indicate the momentum of the particle, calculate the number of microscopic states $\psi(E)$ of this particle.
   (b) If the maximum kinetic energy of the particle is increased to $2E$, is the number of states of the particle increased or decreased? By what factor?
   (c) The kinetic energy of the particle is quantized. Find the expression for possible values of the kinetic energy of this particle (given that the maximum kinetic energy is $E$).

Solutions:

1. (a) total number of microstates: $\psi = 2^4 = 16$
   (b) 5 possible values: $-4\mu$, $-2\mu$, $0$, $2\mu$, $4\mu$
   (c) there are 6 possible combinations that correspond to $M = 0$, so the probability is $6/16 = 37.5\%$

2. (a) the total number of micro-states is the volume of the phase space, which is $VV_{p}$, divided by the volume of the smallest possible cell, which is the minimum of $\Delta x \Delta p_x \Delta y \Delta p_y = h^2$. Or, $\psi = VV_{p}/h^2$. In this 2D problem, $V = L_xL_y$, $V_{p}$ is the volume of
the 2D momentum sphere with a radius of $\sqrt{2mE}$, $V_p = \pi \left( \sqrt{2mE} \right)^2 = 2\pi mE$. Therefore, 

$$V_p = \frac{L_x L_y 2\pi mE}{h^2}.$$ 

(b) When the maximum kinetic energy is increased to $2E$, from the expression in (a), the number of states increases by a factor of 2.

(c) The momentum of the particle is quantized, so is the kinetic energy. Since the particle can be anywhere in the 2D box, $\Delta x = L_x$, $\Delta y = L_y$, and the smallest possible change of the momentum is $\Delta p_x = h/L_x$, $\Delta p_y = h/L_y$. The momentum of the particle can only take the following values: $p_x = n_x \Delta p_x$, and $p_y = n_y \Delta p_y$, where $n_x$ and $n_y$ are integers. The kinetic energy of the particle thus can take the following values:

$$\varepsilon = \frac{p_x^2 + p_y^2}{2m} = \frac{h^2}{2m} \left( \frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} \right).$$ [you are in a good shape if you reached this point!]

Here, $n_x$ and $n_y$ can take the following values: $n_x = 0, 1, 2, \ldots N_x$, $n_y = 0, 1, 2, \ldots N_y$. $N_x$ and $N_y$ are the largest number $n_x$ and $n_y$ can take, which can be determined by the maximum kinetic energy of the particle: $N_x = \frac{\sqrt{2mE}}{\Delta p_x} = \frac{\sqrt{2mEL_x}}{h}$ (when the particle’s momentum is only in the x-direction), and $N_y = \frac{\sqrt{2mE}}{\Delta p_y} = \frac{\sqrt{2mEL_y}}{h}$ (when the particle’s momentum is only in the y-direction).