Physics 425, Spring 2009

Quiz 2 – February 5

Name: ___________________      ID: ___________________

(books and notes are not allowed)

Planck constant: \( h = 6.63 \times 10^{-34} \text{ J s} \), Boltzmann constant: \( k = 1.38 \times 10^{-23} \text{ J/K} \)
Avogadro number: \( N_a = 6.02 \times 10^{23} \)
E = \( \frac{1}{2} f k T \), \( \Delta E = \frac{1}{2} f k \Delta T \), so \( \Delta T = \frac{2 \Delta E}{f k} = 725 \text{ K} \)

For a particular ideal gas system it has been determined that the internal energy is given by \( E = 2.5PV + C \), where \( P \) is the pressure, \( V \) is the volume, and \( C \) is a constant. The system is initially in the state \( P = 2 \times 10^5 \text{ Pa}, V = 0.01 \text{ m}^3 \), and evolves quasi-statically to a final state of \( V = 0.03 \text{ m}^3 \) along the path \( P = 10^5 + 10^9 (V - 0.02)^2 \). (Note: 1 J = 1 Pa \text{ m}^3)

(a) Calculate work \( W \). Is the work done by the system or on the system?
(b) Calculate heat \( Q \). Does the system absorb or release heat?
(c) Suppose that the gas has \( 2 \times 10^{24} \) degrees of freedom, how much does the gas temperature change?

Key:
(a) \( W = \int \frac{P}{V} dV = \int_{V_i=0.01}^{V_f=0.03} \left[ 10^5 + 10^9 (V - 0.02)^2 \right] dV = 10^5 (V_f - V_i) + \frac{10^9}{3} [(V_f - 0.02)^3 - (V_i - 0.02)^3] = 2.67 \times 10^3 \text{ J} \)
W is positive, and also the system is expanding, so the work is done by the system.

(b) \( Q = \Delta E + W \)
\( \Delta E = E_f - E_i = 2.5(P_f V_f - P_i V_i) \)
As \( P_f = 10^5 + 10^9 (V_f - 0.02)^2 = 2.00 \times 10^5 \text{ Pa} \),
\( \Delta E = 2.5(P_f V_f - P_i V_i) = 1 \times 10^4 \text{ J} \)
\( Q = \Delta E + W = 1.27 \times 10^4 \text{ J} \)
\( Q > 0 \), the system absorbs heat to do work as well as increase its internal energy.

(c) \( E = \frac{1}{2} f k T, \Delta E = \frac{1}{2} f k \Delta T \), so \( \Delta T = \frac{2 \Delta E}{f k} = 725 \text{ K} \)