

Physics 425, Spring 2009

Quiz 3 – February 19

Name: _____ ID: _____

(books and notes are not allowed)

Planck constant: $h = 6.63 \times 10^{-34} \text{ J s}$,
Boltzmann constant: $k = 1.38 \times 10^{-23} \text{ J/K}$
Avogadro number: $N_a = 6.02 \times 10^{23}$
Gas constant: $R = 8.31 \text{ J/K/mol}$
 $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$
 $T (\text{K}) = T (^\circ \text{C}) + 273.16$
 $E = f/2 kT$
 $TdS = dE + dW$
 $\log(e) = 0.43$

Note:

- Not all the above constants or relations are useful.
 - Explain how the result is obtained. Generous partial credit will be given to correct steps and explanations even if the final result may be erroneous.
1. A tank has a volume of 0.1 m^3 and is filled with a monatomic ideal gas at a pressure of $6 \times 10^6 \text{ Pa}$ and temperature 200 K . A second tank has a volume of 0.2 m^3 and is filled with the same gas at a pressure of $3 \times 10^6 \text{ Pa}$ and temperature 300 K . A rigid piston separating the two tanks allows thermal interaction but no mechanical interaction or particle transfer between the two tanks. Assuming that the heat transfer between the two tanks is quasi-static, the walls of the tanks are adiabatic and rigid, and the gas has a constant molar specific heat $c_V = 3/2 R$:
- (a) Find the final temperature of the system in final equilibrium [Hint: the internal energy of the combined system is constant.]
 - (b) Find the entropy change of the system. [Hint: the change in the total entropy is the sum of entropy change in each sub system.]
 - (c) From the initial to the final state, by what factor does the number of quantum states change? Express your answer in the powers of ten.

Solution :

(a) In final equilibrium, the gases have the same temperature T_f .

$$\text{The constant internal energy is } E = \frac{3}{2} N_1 k T_1 + \frac{3}{2} N_2 k T_2 = \frac{3}{2} (N_1 + N_2) k T_f$$

Using the ideal gas law : $PV = NkT$, there are :

$$N_1 T_1 k = P_1 V_1, N_2 T_2 k = P_2 V_2, N_1 k = \frac{P_1 V_1}{T_1}, N_2 k = \frac{P_2 V_2}{T_2}$$

$$\text{Therefore, } T_f = \frac{N_1 T_1 + N_2 T_2}{N_1 + N_2} = \frac{\frac{P_1 V_1}{T_1} + \frac{P_2 V_2}{T_2}}{\frac{P_1 V_1}{T_1} + \frac{P_2 V_2}{T_2}} = 240 \text{ K}$$

(b) Using the second law $dS = (dE + PdV)/T = \frac{C_V dT}{T} + \frac{PdV}{T}$

Here the volumes of the two tanks are fixed, so $dV = 0$,

$$\text{thus } dS = \frac{C_V dT}{T} = \frac{nc_V dT}{T}, \text{ yielding } \Delta S = nc_V \ln(T_f / T_i)$$

Using $PV = nRT$, there is $n_1 = \frac{P_1 V_1}{RT_1}, n_2 = \frac{P_2 V_2}{RT_2}$. Therefore,

$$\Delta S = \Delta S_1 + \Delta S_2 = \frac{3}{2} \frac{P_1 V_1}{T_1} \ln(T_f / T_1) + \frac{3}{2} \frac{P_2 V_2}{T_2} \ln(T_f / T_2) = 151 \text{ J/K}$$

$$(c) \frac{\Omega_f}{\Omega_i} = e^{\frac{\Delta S}{k}} = e^{\frac{151 \text{ J/K}}{1.38 \times 10^{-23} \text{ J/K}}} = e^{1.1 \times 10^{25}} = 10^{4.8 \times 10^{24}}$$