

Physics 425, Spring 2009

Quiz - 5
April 2, Thursday

Name: _____ ID: _____

(books and notes are not allowed)

Planck constant: $h = 6.63 \times 10^{-34} \text{ J s}$,

Boltzmann constant: $k = 1.38 \times 10^{-23} \text{ J/K}$

Avogadro number: $N_a = 6.02 \times 10^{23}$

Gas constant: $R = 8.31 \text{ J/K/mol}$

$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$

$T (\text{K}) = T (^{\circ} \text{C}) + 273.16$

$\log(e) = 0.43$

Note:

- Not all the above constants or relations are useful.
- Explain how the result is obtained. Generous partial credit will be given to correct steps and explanations even if the final result may be erroneous.

1. The nucleus of an atom in a certain solid can be in any one of three spin states labeled by the quantum number m , where $m = 1, 0$, or -1 . This quantum number measures the projection of the nuclear spin along a crystal axis of the solid. In the spin states $m = 1$ and $m = -1$, the nucleus has the same spin energy $\varepsilon > 0$, and in the spin state $m = 0$, the nucleus spin energy is zero.
 - (a) The solids are kept in thermal equilibrium with a constant temperature T . Find the number of atoms with spin $m = 1$ relative to the number of atoms with spin $m = 0$.
 - (b) If $\varepsilon = 0.1 \text{ eV}$, at what temperature, one half of the atoms in the solid have spin $m = 0$?
 - (c) Qualitatively discuss what is the mean energy per atom at very high temperature and at very low temperature. Please give physical explanation of your conclusion.
 - (d) Find the exact expression of the mean energy per atom in this solid.
 - (e) From the result in (d), verify your conclusions in (c).

Solution;

(a) The probability of the nucleus in a certain spin state is given by $P_m = Ce^{-\varepsilon_m/kT}$.

So the number of particles in spin $m = 1$ relative to the number of particles in spin state

$$m = 0 \text{ is given by } \frac{N_1}{N_0} = \frac{P_1}{P_0} = e^{-(\varepsilon_1 - \varepsilon_0)/kT} = e^{-\varepsilon/kT}$$

(b) The probability of particles in spin state $m = 0$ is given by $P_0 = \frac{e^{-\varepsilon_0/kT}}{e^{-\varepsilon_0/kT} + e^{-\varepsilon_{-1}/kT} + e^{-\varepsilon_1/kT}}$.

$$\text{Since } \varepsilon_0 = 0, \varepsilon_1 = \varepsilon_{-1} = \varepsilon, P_0 = \frac{1}{1 + 2e^{-\varepsilon/kT}}. \text{ If } P_0 = \frac{1}{1 + 2e^{-\varepsilon/kT}} = 1/2, \text{ then } T = \frac{\varepsilon}{k \ln 2} = \frac{0.1 \times 1.6 \times 10^{-19} \text{ J}}{1.38 \times 10^{-23} \ln 2 \text{ J/K}} = 1673 \text{ K.}$$

(c) At very low temperature, the system does not have enough thermal energy to get to the excited states at higher energies, so the system stays at the ground state with the lowest energy. Therefore, as T approaches zero, the mean energy is approaching the $m = 0$ state which has the lowest energy 0. At very high temperatures, the system has nearly equal probability to stay in any m state, because the Boltzmann factor all tend to 1. Then the mean energy per particle is the mean of energies of all state: $\bar{\varepsilon} = \frac{\varepsilon_0}{3}$.

$$(d) \bar{\varepsilon} = \sum P_m \varepsilon_m = \frac{2\varepsilon e^{-\varepsilon/kT}}{1 + 2e^{-\varepsilon/kT}}$$

(e) At very low temperature, ε/kT approaches infinity, and the exponential function $e^{-\varepsilon/kT}$ approaches zero. So

$$\bar{\varepsilon} \approx \frac{2\varepsilon 0}{1 + 0} = 0, \text{ consistent with the argument in (c).}$$

At very high temperature, $\varepsilon/kT \approx 0$, so the exponential function $e^{-\varepsilon/kT} \approx 1$, and $\bar{\varepsilon} \approx \frac{2\varepsilon}{1 + 2} = \frac{2\varepsilon}{3}$.