

### III.3 Maxwell distribution

class schedule: Apr 6

reading assignments: S16A-B, R7.9-7.10

#### III.3.1 Distribution of velocity

The **velocity and speed distributions** of gas particles is usually treated in a classical approach. The number density distribution of particles with a velocity component between  $v_x$  and  $v_x + \delta v_x$  is given by

$$f(v_x)dv_x = n\left(\frac{m}{2\pi kT}\right)^{1/2}e^{-mv_x^2/2kT}dv_x$$

where  $n$  is the particle number density  $n = N/V$ .  $\left(\frac{m}{2\pi kT}\right)^{1/2}$  is a normalization factor so that  $\int_{-\infty}^{\infty} f(v_x)dv_x = n$ . As there is equal chance for a particle to have a velocity of  $v_x$  and  $-v_x$ ,  $f(v_x)$  is symmetric about  $v_x = 0$ , and the mean  $v_x$  is zero. By the Boltzmann factor, it is seen that the distribution function peaks at  $v_x = 0$ .

The number density distribution of particles with velocities in the range of  $v_x$  and  $v_x + \delta v_x$ ,  $v_y$  and  $v_y + \delta v_y$ , and  $v_z$  and  $v_z + \delta v_z$  is the product of the three individual number densities:

$$f(v_x, v_y, v_z)dv_x dv_y dv_z = n\left(\frac{m}{2\pi kT}\right)^{3/2}e^{-m(v_x^2+v_y^2+v_z^2)/2kT}dv_x dv_y dv_z$$

#### III.3.2 Distribution of speed

The number density distribution of particles with its speed between  $v$  and  $v + dv$  can be also derived as

$$F(v)dv = 4\pi n\left(\frac{m}{2\pi kT}\right)^{3/2}v^2e^{-mv^2/2kT}dv$$

where  $v^2 = v_x^2 + v_y^2 + v_z^2$ , and  $\int_0^{\infty} F(v)dv = n$ . The  $4\pi v^2$  in the integral apart from the Boltzmann factor is to take care of the fact that particles in a momentum volume of a spherical shell with an area  $4\pi v^2$  and thickness  $dv$  all have the speed between  $v$  and  $v + dv$ . In other words,  $4\pi v^2 dv$  term takes care of particles at a certain speed moving in all directions. Microscopically, a number of micro-states have the same kinetic energy  $\frac{1}{2}mv^2$ , and the number of micro-states is proportional to  $v^2$ . When counting the number of particles, or probabilities, in a given energy range, the distribution function has to be multiplied by the number of micro-states assuming the same energy.

Example 1: the **most probable speed**, **mean speed**, and **root-mean-square speed** are calculated using the speed distribution function  $F(v)$ .

Example 2: evaluating mean values using the Maxwell distribution

Example 3: thermal and non-thermal distribution of gas particles.

Example 4: Doppler broadening of spectral lines

Example 5: an intuitive way to derive the expression for  $P$  for an ideal gas.