

### III.4 Paramagnetism

class schedule: Apr 8

reading assignments: S17, R7.8

examples marked with \* are important and should be studied carefully.

A charged particle with a spin or orbit motion generates a loop current, which can interact with an external magnetic field. The magnetic moment of such a particle is proportional to the component of its angular momentum of the spin or orbit motion along the direction of the external magnetic field, supposed to be along  $z$  direction.

For orbiting particles,  $\mu_z = \frac{ge}{2m}L_z$ , where  $e$  is the electric charge,  $g$  is the Lande  $g$  factor,  $m$  is the mass of the particle, and  $L_z$  is the angular momentum along  $z$  direction.  $L_z$  is quantized and can only take values  $L_z = l_z\hbar$ , where  $l_z$  is the quantum number of angular momentum along  $z$  direction,  $l_z = 0, \pm 1, \pm 2, \dots \pm l$ , and  $l$  is the quantum number of the angular momentum of the particle. For protons, neutrons, and electrons,  $g = 1, 0, -1$ , respectively.

Example 1: magnetic moment for protons and electrons: Bohr magneton and nuclear magneton.

For spin particles,  $\mu_z = \frac{ge}{2m}S_z$ , where  $S_z$  is the angular momentum, which can only take values of  $\pm \frac{1}{2}\hbar$  for protons, neutrons, and electrons.  $g = -2, -3.82, 5.58$  for electrons, neutrons, and protons, respectively.  $g$  values reflect mass and charge distribution in a spin particle.

Example 2: magnetic moment in an atom: since nuclear particles (protons and neutrons) are a lot heavier than electrons, the magnetic moment of an atom is mostly contributed by spinning and orbiting electrons, so that  $\mu_z \approx -\mu_B(l_z + 2s_z)$ , where  $\mu_B = \frac{e\hbar}{2m}$  is the Bohr magneton and  $l_z$  and  $s_z$  are quantum numbers specifying the angular momenta of electrons' orbit and spin motion along  $z$  direction, i.e., the external  $\mathbf{B}$  direction.

Consider the paramagnetism due to the presence of an unpaired electron with spin 1/2, which can only be parallel or anti-parallel to the external field  $\mathbf{B}$ . For the parallel spin, the magnetic moment  $\mu_+ = \mu_B$  and the magnetic energy  $\epsilon_+ = -\mu_+B = -\mu_B B$ . For anti-parallel spin, the magnetic moment  $\mu_- = -\mu_B$  and the magnetic energy  $\epsilon_- = -\mu_-B = \mu_B B$ . The mean atomic magnetic moment is then given by

$$\bar{\mu} = P_+\epsilon_+ + P_-\epsilon_- = \mu_B \tanh \frac{\mu_B B}{kT}$$

The mean magnetization is the mean magnetic moment per mole,  $M = N_a \bar{\mu}$ .

\*Example 3: paramagnetism and Curie law: when  $\mu_B B/kT \gg 1$ ,  $\bar{\mu} \approx \mu_B$ , and  $M \approx N_a \mu_B$ , independent of the external field  $B$  or temperature  $T$ , or the magnetization is saturated at the low temperature limit. When  $\mu_B B/kT \ll 1$ ,  $\bar{\mu} = \frac{\mu_B^2 B}{kT}$  and  $M = \frac{N_a \mu_B^2 B}{kT}$ , which is inversely proportional to  $T$ . This is known as **Curie law**.

\*Example 4 (Stowe 17C, Reif 7.8): general calculation of magnetization:  $\epsilon = -\mu B = -\mu_B j_z B$ , where the external magnetic field  $\mathbf{B}$  is along  $z$  direction and  $j_z = 0, \pm 1, \pm 2, \dots \pm j$  is the quantum

number of the total angular momentum along the magnetic field or  $z$  direction. The mean magnetic moment over all  $j_z$  values becomes  $\bar{\mu} = \mu_B[(j + \frac{1}{2})\coth(j + \frac{1}{2}\eta) - \frac{1}{2}\coth\frac{1}{2}\eta]$ , where  $\eta = \frac{\mu_B B}{kT}$ . The magnetization  $M = N_a \bar{\mu}$ . When  $\eta \ll 1$ ,  $M \sim B/T$ , the Curie law. At the low temperature limit,  $M \approx N_a \mu_B j$ , reaching the saturation independent of  $B$  and  $T$ . It is found that at  $1.3^0$  K, an external magnetic field of 50000 Gauss can produce more than 99.5% magnetic saturation in some metals.

Note that the above discussion on the *intrinsic magnetism* of materials ignores the interaction between particles and treats  $\mathbf{B}$  as entirely the external field.