

III.6 Photon statistics: blackbody radiation

Class schedule: April 16

Reading assignments: S21, R9.13; optional: S20

III.4.2 Blackbody Radiation

Photons are Bosons. Specifically for photons, $\mu = 0$. The photon energy is quantized $\epsilon(\omega) = \hbar\omega$, where ω is the frequency of the electromagnetic wave, with $\omega = \lambda c$ in a wave train, λ being the wavelength, and c the speed of light. Therefore, the **photon statistics**, or the BE distribution with zero μ , can be written as

$$\bar{n}_\omega = \frac{1}{e^{\hbar\omega/kT} - 1}$$

The electromagnetic radiation in an opaque object in thermal equilibrium (like an oven or a star) can be treated as a photon gas. The radiation energy density per unit frequency or wavelength is the total photon energy integrated in the momentum space, written as:

$$du(\omega) = \frac{\hbar}{\pi^2 c^3} \frac{\omega^3 d\omega}{e^{\hbar\omega/kT} - 1}$$

or

$$du(\lambda) = \frac{8\pi hc}{\lambda^5} \frac{d\lambda}{e^{hc/\lambda kT} - 1}$$

We often use another quantity, the **radiation flux**, or energy per unit area per unit time per unit solid angle per unit frequency or wavelength, defined by

$$F(\omega, T) = \frac{c}{4\pi} \frac{du(\omega)}{d\omega} = \frac{\hbar}{4\pi^3 c^2} \frac{\omega^3}{e^{\hbar\omega/kT} - 1}$$

or

$$F(\lambda, T) = \frac{c}{4\pi} \frac{du(\lambda)}{d\lambda} = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$$

This is the **Planck's function** for **blackbody radiation**.

The Planck's function suggests that at higher temperature, the radiation energy density, or average photon energy, or radiation flux, is greater, and the radiation is more toward the short-wavelength range, i.e., bluer. The total radiation energy and radiation flux integrated over all frequencies/wavelengths is proportional to T^4 .

Example 1: Rayleigh-Jean's law and ultraviolet catastrophe (HW)

Example 2: the Wien's displacement law $\lambda_{max} T = 2.9 \times 10^{-3}$ m K, the Stefan-Boltzmann's law $F(T) = \sigma T^4$ Jm⁻²s⁻¹, $\sigma = 5.67 \times 10^{-8}$ Jm⁻²s⁻¹K⁻⁴, and the observational Hertzsprung-Russell diagram for main-sequence stars.

Example 3: radiation pressure