III.6 Photon statistics: blackbody radiation

Class schedule: April 16

Reading assignments: S21, R9.13; optional: S20

III.4.2 Blackbody Radiation

Photons are Bosons. Specifically for photons, $\mu = 0$. The photon energy is quantized $\epsilon(\omega) = \hbar \omega$, where $\omega$ is the frequency of the electromagnetic wave, with $\omega = \lambda c$ in a wave train, $\lambda$ being the wavelength, and $c$ the speed of light. Therefore, the photon statistics, or the BE distribution with zero $\mu$, can be written as

$$\bar{n}_\omega = \frac{1}{e^{\hbar \omega / kT} - 1}$$

The electromagnetic radiation in an opaque object in thermal equilibrium (like an oven or a star) can be treated as a photon gas. The radiation energy density per unit frequency or wavelength is the total photon energy integrated in the momentum space, written as:

$$du(\omega) = \frac{\hbar}{\pi^2 c^3} \frac{\omega^3 d\omega}{e^{\hbar \omega / kT} - 1}$$

or

$$du(\lambda) = \frac{8\pi \hbar c}{\lambda^5} \frac{d\lambda}{e^{\hbar c/\lambda kT} - 1}$$

We often use another quantity, the radiation flux, or energy per unit area per unit time per unit solid angle per unit frequency or wavelength, defined by

$$F(\omega, T) = \frac{c}{4\pi} \frac{du(\omega)}{d\omega} = \frac{\hbar}{4\pi^3 c^2} \frac{\omega^3}{e^{\hbar \omega / kT} - 1}$$

or

$$F(\lambda, T) = \frac{c}{4\pi} \frac{du(\lambda)}{d\lambda} = \frac{2\hbar c}{\lambda^5} \frac{1}{e^{\hbar c/\lambda kT} - 1}$$

This is the Planck’s function for blackbody radiation.

The Planck’s function suggests that at higher temperature, the radiation energy density, or average photon energy, or radiation flux, is greater, and the radiation is more toward the short-wavelength range, i.e., bluer. The total radiation energy and radiation flux integrated over all frequencies/wavelengths is proportional to $T^4$.

Example 1: Rayleigh-Jean’s law and ultraviolet catastrophe (HW)

Example 2: the Wien’s displacement law $\lambda_{\text{max}} T = 2.9 \times 10^{-3}$ m K, the Stefan-Boltzmann’s law $F(T) = \sigma T^4 \text{Jm}^{-2}\text{s}^{-1}$, $\sigma = 5.67 \times 10^{-8}\text{Jm}^{-2}\text{s}^{-1}\text{K}^{-4}$, and the observational Hertzsprung-Russell diagram for main-sequence stars.

Example 3: radiation pressure