

III.8 Conduction Electrons in Metals

Class schedule: Apr 23, Apr 28

Reading assignment: S20D, S22C-D, R9.16

Examples marked with * are important.

Electrons in metals can be considered to be free particles forming an electron gas. Electrons are 1/2 spin particles, or Fermions, following the Fermi-Dirac distribution

$$\bar{n}_s = \frac{1}{e^{(\epsilon_s - \mu)/kT} + 1}$$

where μ is called the **Fermi level**.

At the Fermi energy, $\bar{n}_s = 1/2$. When T approaches zero, all states with energy below the Fermi level are occupied, each by one electron, and all states with energy above Fermi level are empty. In this case, we can calculate the Fermi level $\mu(T = 0)$, which is called the **Fermi energy**, by the integral $\int g(\epsilon)n(\epsilon)d\epsilon = N$. Particularly, $g(\epsilon)$ takes the nonrelativistic form, and the integral covers from 0 to $\mu(0)$ where $n(\epsilon) = 1$.

*Example 1: the **Fermi energy**, **Fermi temperature**, and total kinetic energy of conduction electrons in metals at $T = 0$:

$$\epsilon_F \equiv \mu(0) = \frac{h^2}{2m} \left(\frac{3}{8\pi} \frac{N}{V} \right)^{2/3}, T_F \equiv \frac{\epsilon_F}{k}, E = N \frac{3}{5} \epsilon_F$$

Even at $T = 0$, for ordinary solids, the Fermi temperature is of order 10^{4-5} K, so the electron kinetic energy is very large, a few orders of magnitude larger than gas particles at the room temperature. Because of the very high Fermi temperature, at the room temperature, conduction electrons are almost all frozen into the Fermi well. The total energy of the Fermion gas does not change significantly with the temperature, so conduction electrons have very trivial contribution to the specific heat of the metal.

Example 2: specific heats of solids: insulators and conductors: only at very low temperatures, conduction electrons contribute to the specific heat of the solid as proportional to T , which can be significant compared with the specific heat associated with the lattice vibration, which is proportional to T^3 (phonon gas).

*Example 3 (review): specific heats of gas and solids: all things considered (HW).

Example 4: gas pressure, radiation pressure in the stellar interior, and electron degeneracy pressure in white dwarfs.