

## II.7 Application: Heat Engines and Refrigerators

Reading assignment: S13, R5.11-12, F7-10

Class schedule: March 12, 24

### II.7.1 Heat Engines

Ideally a perfect heat engine would extract an amount of heat  $Q$  from a source, or a heat reservoir, and convert the entire amount of  $Q$  into work  $W$ , while the heat engine comes back to its initial state (to be able to perform the next cycle) while causing no other changes in the environment. This scenario, however, violates the second law.

Example 1: the Kelvin postulate and the "perpetuum mobile" of the second kind.

To abide by the second law, an engine can convert a part of heat  $Q_1$  absorbed from a heat reservoir at  $T_1$  into work  $W$  while releasing the rest amount  $Q_2 = Q_1 - W$  to another heat reservoir at lower temperature  $T_2$ .

Example 2: calculate the efficiency of a heat engine:

$$\eta = \frac{W}{Q_1} \leq \frac{T_1 - T_2}{T_1} = \eta_{Carnot}$$

The Carnot cycle depicts a quasi-static cyclic process of such a heat engine, consisting of 4 stages to complete a cycle between  $T_1$  and  $T_2$ , with  $T_1 > T_2$ .

- isothermal expansion at  $T_1$ , absorbing heat  $Q_1$  and doing work (ignition cycle)
- adiabatic expansion to  $T_2$ , doing work (work cycle)
- isothermal compression at  $T_2$ , releasing heat  $Q_2$  and receiving work (exhaust cycle)
- adiabatic compression to  $T_1$ , receiving work (compression cycle)

Example 3: calculate  $Q$  and  $W$  in the Carnot cycle from a P-V diagram for an ideal gas and a van der Waals gas.

### II.7.2 Refrigerators

A perfect refrigerator would extract an amount of heat  $Q$  from a source  $T_2$  and releases the entire amount of  $Q$  to another heat reservoir  $T_1$ , where  $T_1 > T_2$ . This scenario also violates the second law.

Example 4: Clausius postulate, definition of absolute temperature and direction of heat flow.

Remark: historically, the postulates of Kelvin and Clausius led to the second thermodynamic law and the thermodynamic definition of entropy.

A perfect refrigerator not violating the second law extracts an amount of heat  $Q_2$  from  $T_2$  and requires work  $W$  to release a total amount of heat  $Q_1 = Q_2 + W$  to  $T_1$ . This can be done by reversing the Carnot cycle: adiabatic expansion to  $T_2$ , isothermal expansion at  $T_2$  absorbing  $Q_2$ , adiabatic compression to  $T_1$ , and isothermal compression at  $T_1$  releasing  $Q_1$ .

### II.7.3 Reversible and Irreversible Processes

Per the second law, for an isolated system, a process is reversible only when the net change in entropy  $\Delta S$  is zero. Otherwise, the reversed process may be achieved (i.e., with decreased entropy) by interacting with external systems but at the cost of increased entropy in interacting systems.

Example 5: free expansion of a gas is an irreversible process. For an ideal gas, the internal energy remains constant, and increased volume will significantly increase the number of accessible quantum states, or entropy, as  $\Omega \sim V^N$ .

Example 6: irreversibility of thermal conduction: tiny deviation from thermal equilibrium, the maximum entropy state, will result in decreased number of micro-states by the factor of  $e^{-N(\delta E/E)^2}$ !

Adiabatic process and reversibility: a **quasi-static adiabatic process** results in zero entropy change and is hence reversible.  $\Delta S > 0$  for an adiabatic process which is NOT quasi-static. On the other hand, non-adiabatic quasi-static process can be reversible when the net change of  $\Delta S = \int dS = \int \frac{dQ}{T}$  is zero, such as in the Carnot cycle.

Example 7: the Carnot cycle is a reversible process. Any realistic engine ends up with increased entropy thus  $\eta < \eta_{Carnot}$ .