Solar flares as broken flux constraints in active region magnetic modeling

Lucas Tarr
MSU Solar Physics
Jan. 24, 2013
1. Modeling a single time
2. Modeling field evolution
3. Gettin the ergs
4. Break that constraint!
Modeling a single time
Modeling field evolution
Gettin the ergs
Break that constraint!

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Bozeman, Jan 24, 2013
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Break that constraint!
A little magnetostatic reminder:

\[-\nabla P + \frac{1}{4\pi} (\nabla \times \mathbf{B}) \times \mathbf{B} - \rho \nabla \Phi_g = 0\]
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if \( \frac{B^2}{4\pi L} \gg \frac{P}{L}, \frac{\rho \Phi_g}{L} \), then the Lorentz force is self balancing:

\[(\nabla \times \mathbf{B}) \times \mathbf{B} = 0\]

or \( \Rightarrow \nabla \times \mathbf{B} = \alpha \mathbf{B} \)
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Options: \(\alpha = \alpha_0 \rightarrow \text{LFFF},\)
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Also, \(\nabla \cdot \mathbf{B} = -\nabla^2 \Phi = 0\)
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Also also, \(\frac{4\pi}{c} \mathbf{J} = \nabla \times \mathbf{B} = 0\): No current.
Modeling a single time evolution field, Gettin' the ergs.

Break that constraint!

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Bozeman, Jan 24, 2013
Modeling a single time model evolution. Getting the ergs. Break that constraint!
Modeling a single time Modeling field evolution Gettin the ergs Break that constraint!

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Modeling a single time

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Domains containing flux in the potential field configuration
Modeling the whole field

\[ \psi_{i+1}^j = \psi_i^j + \sum_k \Delta_i S_{j,k} \]

Surface change matrix between times \( i \) and \( i + 1 \):

\[ \Delta_i S_{j,k} \]

I wrote a simple algorithm to deal with this guy

Then, you just stack those changes together over time, and you have the flux in any pole at time \( i \) in terms of its (sub)emergence history with all other regions:

\[ \psi_i^j = \psi_0^j + \sum_{l=0}^{i-1} \sum_k \Delta_l S_{j,k} \]

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Modeling the whole field... starting with just the total flux in a single pole:

\[ \psi_{j}^{i+1} = \psi_{j}^{i} + \sum_{k} \Delta^{i} S_{j,k} \]

flux at some time = previous flux + change at boundary
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\[ \psi_{j}^{i} = \psi_{j}^{0} + \sum_{l=0}^{i-1} \sum_{k} \Delta^{l}S_{j,k} \]
Regions with which P3 has emerged:

Using a cool symbol:

$$\psi_{P3,N*}(i) = \sum \Delta^i \mathbb{S}$$
Same expression, now for all domains: drop the subscripts. The “real” field’s connectivity $F$ at any time $i$ is

$$F^i = P^0 + \sum_{j=0}^{i-1} \Delta^j S$$
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As stated before, the separators force the domain fluxes into configuration $F$ instead of a potential configuration $P$. This is equal to the self–flux of the current loop:

$$\psi^{(cr)}_\sigma i = \psi^i_\sigma - \psi^{(v)}_\sigma i = \sum_D F^i_D - \sum_D P^i_D$$

with $D$ the domains which force separator $\sigma$ to have currents.
This is a matrix of how the flux has been redistributed relative to the potential field: \( P^i = F^i + R^i \)
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... which we call \( R \)

\[
\psi^i_{(\text{cr})} = \psi^i_\sigma - \psi^i_{(\nu)} = \sum_D F^i_D - \sum_D P^i_D
\]

\[
\equiv - \sum_D \sum_{j=0}^{i-1} \Delta^j R_D.
\]
P3’s elements in $\mathbb{R}$; or, how much flux must you add to each of P3’s domains to make it potential?
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The non potentiality of these domains affect 8 of the 23 separators at the M2.2 flare time.
Longcope & Magara (2004) did some heavy lifting to analytically relate each separator’s current to that self-flux:

\[ \psi^{(cr)i} \equiv \psi^i - \psi^{(v)i} = \frac{IL}{4\pi} \ln \left( \frac{el^*}{|I|} \right) \]

So, we have a current, and we have a flux relative to a potential flux, so we can calculate an energy:
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So, we have a current, and we have a flux relative to a potential flux, so we can calculate an energy:

\[
\Delta W_{MCC}(I) = \frac{1}{4\pi} \int_{\psi_{potl}}^{\psi} Id\psi = \frac{LI^2}{32\pi^2} \ln \left( \frac{\sqrt{el^*}}{|I|} \right)
\]
Invert the current/separat–flux relation:

$$I(\psi^{(cr)i}_\sigma) = I^* \Lambda^{-1}(4\pi \psi^{(cr)i}_\sigma / LI^*)$$

with

$$\Lambda(x) \equiv x \ln(e/|x|)$$
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...and solve for the work:

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Invert the current/seperator-flux relation:

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...and solve for the work:

\[ \Delta W_{MCC}(I) \longrightarrow \Delta W_{MCC}(\psi_{\sigma}^{(cr)i}) \]

But from a couple slides ago, that we have an expression for that!

\[ \psi_{\sigma}^{(cr)i} \equiv -\sum_{D} \sum_{j=0}^{i-1} \Delta^j R_D. \]
Separator fluxes and the corresponding free energies: various constraint times compared to flare–time potential field

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Flares are things that break the flux constraints... so let’s break them!
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Plan: minimize the separator currents at a flare time $k$ by adding on a flux redistribution $\mathbf{X}^k$ (AKA reconnection)

$$\mathbf{F}_{\text{postflare}}^k = (\mathbf{F}_{\text{preflare}}^k + H(k)\mathbf{X}^k)$$
Flares are things that break the flux constraints... so let’s break them!
Plan: minimize the separator currents at a flare time $k$ by adding on a flux redistribution $X^k$ (AKA reconnection)

$$F_{\text{postflare}}^k = (F_{\text{preflare}}^k + H(k)X^k)$$

This gives you the opposite change in $R$. If you have multiple flares at times $k, l, m, \ldots$ just stack them together:

$$\psi^{(cr)i}_\sigma = - \sum \sum_{D} \sum_{j=0}^{i-1} \Delta^j R_D$$

$$+ \sum_D \left( H(k)X^k_D + H(l)X^l_D + \ldots \right) .$$
Here’s a summary table:

<table>
<thead>
<tr>
<th>Flare</th>
<th>Flux (Mx)</th>
<th>Domains</th>
<th>Separators</th>
<th>Steps</th>
</tr>
</thead>
<tbody>
<tr>
<td>M6.6</td>
<td>42357</td>
<td>10</td>
<td>3</td>
<td>1922</td>
</tr>
<tr>
<td>M2.2</td>
<td>19977</td>
<td>15</td>
<td>8</td>
<td>327</td>
</tr>
<tr>
<td>X2.2</td>
<td>210125</td>
<td>17</td>
<td>10</td>
<td>29504</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Initial $E_{\text{MCC}}$</th>
<th>$\Delta E_{\text{MCC}}$</th>
<th>$E_{\text{potl}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>M6.6  $1.53 \times 10^{32}$</td>
<td>$3.89 \times 10^{30}$</td>
<td>$3.83 \times 10^{32}$</td>
</tr>
<tr>
<td>M2.2  $1.65 \times 10^{32}$</td>
<td>$2.62 \times 10^{30}$</td>
<td>$5.77 \times 10^{32}$</td>
</tr>
<tr>
<td>X2.2  $2.94 \times 10^{32}$</td>
<td>$1.68 \times 10^{32}$</td>
<td>$5.55 \times 10^{32}$</td>
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</tbody>
</table>

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I’ve done that at 4 times, and got the world’s most chunky figure out of it:
Upshot: you can study active–region evolution timescales with topological models, informed by real data

Still not so hot for flare prediction, but...

Could help explain why sometimes different flares involve different portions of a single active region

Taken with some caution, it’s encouraging that this very different method of energy calculation gives comparable methods to, say, NLFFF
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The End.