Fourier Modeling of DEM Distributions

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Outline of Talk

I. DEM Techniques
   A. Limitations of filter ratios
   B. DEM curve limitations
II. Fourier Coefficient Method
   A. Representing a DEM curve as a Fourier expansion
   B. Definition of error bars
III. Examples and Proof of Concept
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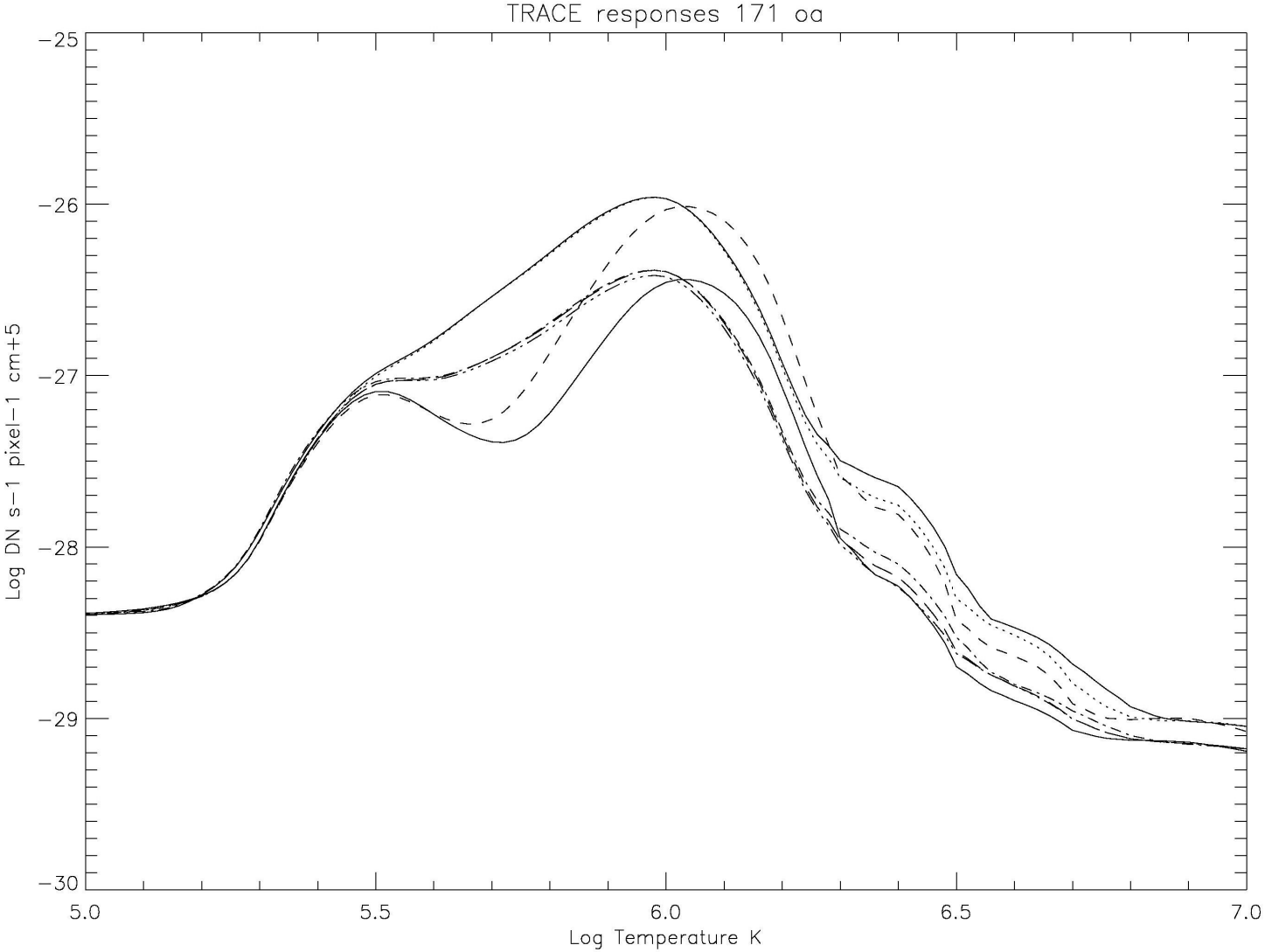
   B. SEMAST vs. Forward Modeling
IV. Results From The 18_sep_2001 Loop Analysis
   A. Lines used
   B. SEMAST vs. CHIANTI
IV. Conclusions

DEM Techniques
Limitations of Filter Ratios

- Assumes an isothermal distribution automatically
- "The Inadequacy of Temperature Measurements in the Solar Corona through Narrowband Filter and Line Ratios", Martens, P. C. H., Cirtain, J. W., Schmelz, J. T. 2002, ApJ., 577, 2, pp. L115-L117
- Different coronal assumptions give you different answers!

TRACE 171 Temp. Response Under Different Coronal Assumptions



DEM Techniques

DEM Curve Limitations

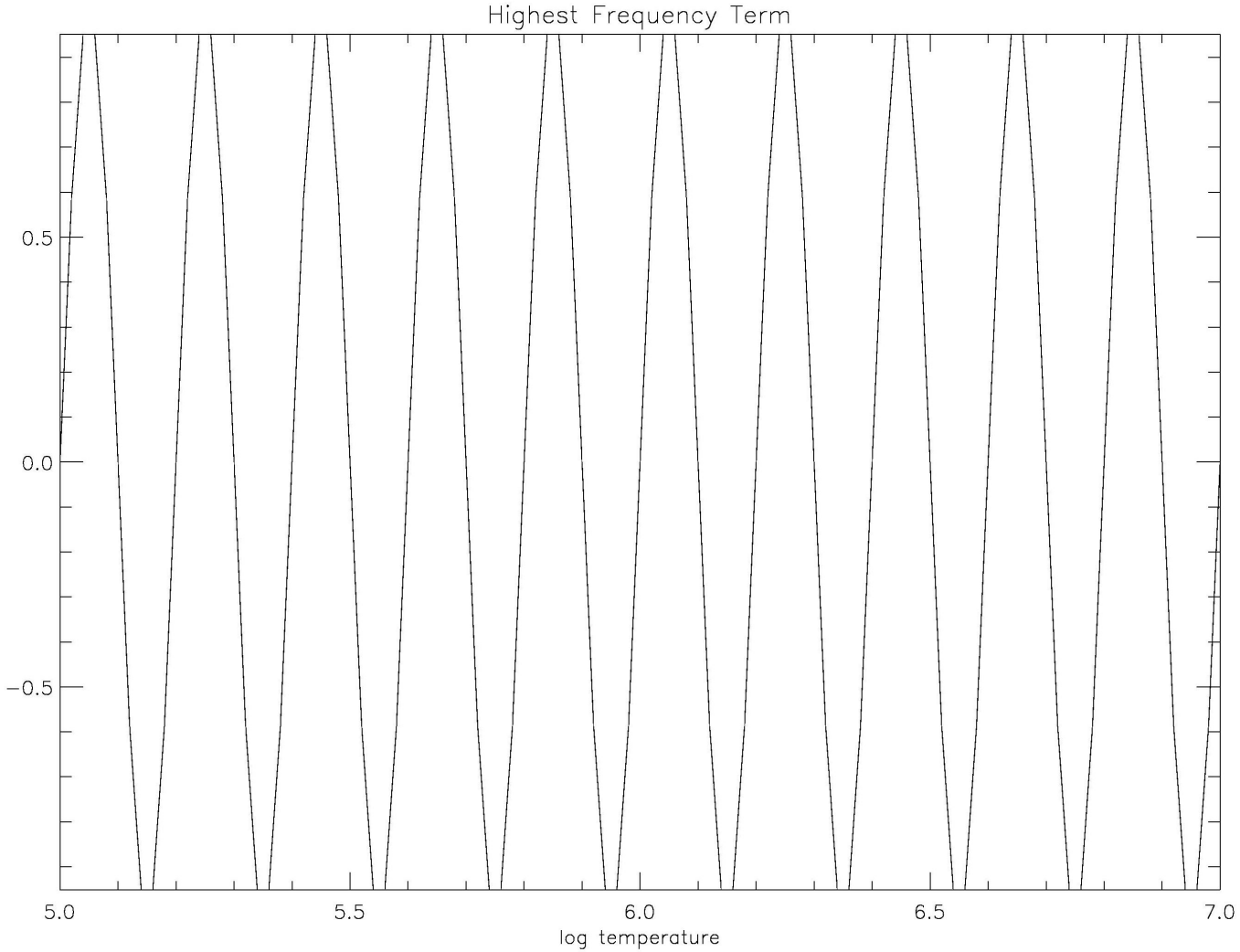
- DEM solutions are ill-posed Fredholm Equations of the First Kind (notoriously hard to solve)
- Have to make assumptions about the solutions: continuous, non_negative, etc.
- Smoothness of DEM???
  - Quantify please???
  - Physical Source of Smoothness??? Or just Mathematics?
- Different coronal assumptions give you different answers, still!!!

Representing a DEM curve as a Fourier expansion

$$f(t)=a_0 + a_1\sin(1*t) +\ldots + b_1\cos(1*t) +\ldots$$

- All inversion techniques have to solve for some set of parameters to represent the DEM curve.
  - The fewer the better (Hopefully n-1 at least!!! Forward modeling often violates this statistical rule.)
- By using Fourier expansions one can describe a complex curve using only a few coefficients. (Well within n-1)
- Continuous smooth functions are assured since it just a superposition of sines and cosines without artificial smoothing, just a finite number of coefficients.

Highest Frequency Term of an Expansion Containing 11 Spectral Lines



Solving for a DEM Curve as a Fourier Expansion

This is the process used by the Solar Emission Measure Analysis Software Toolkit (SEMAST) in order to solve for DEM curves

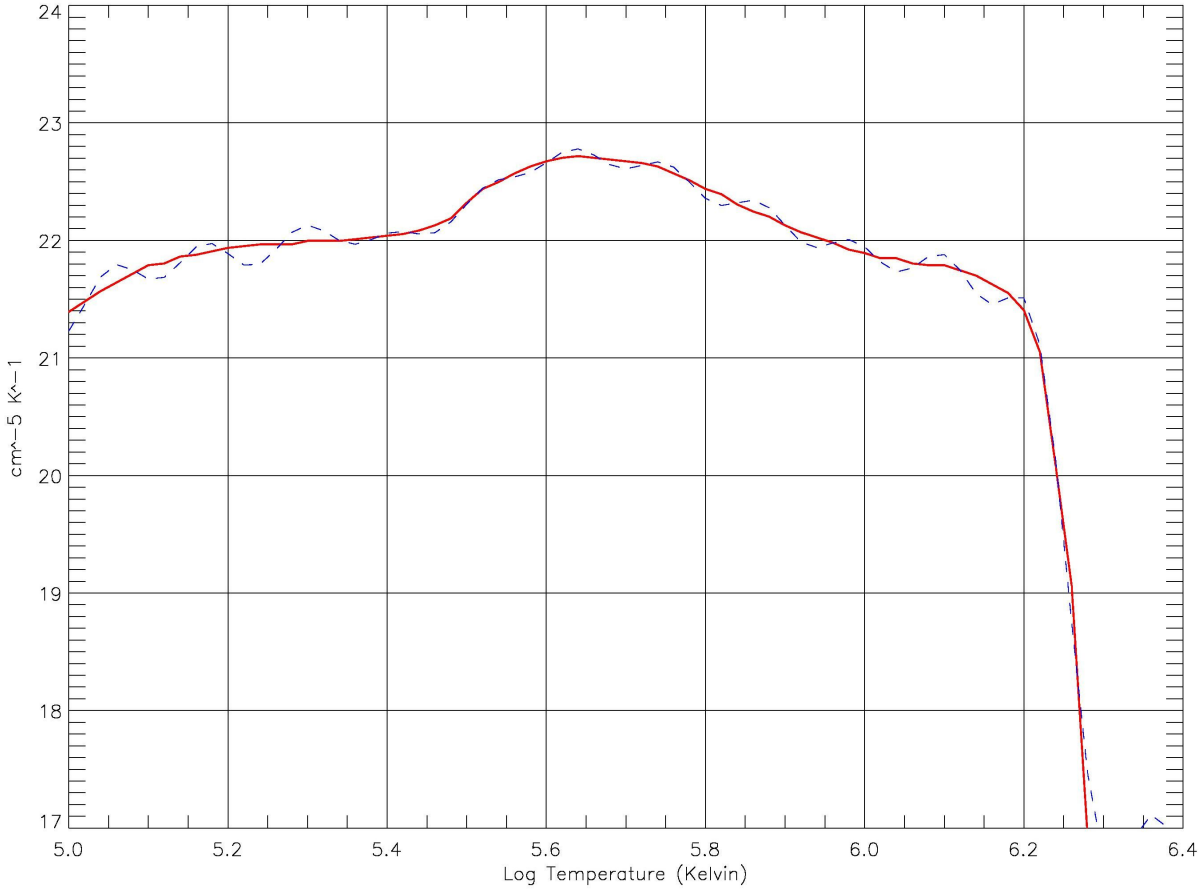
- Apply proper background subtraction to observed intensities
- Generate G of T functions for the observed lines under varying coronal conditions using the CHIANTI atomic physics database. (ADAS and APEC can also be used.)
- Assumptions/Constraints on DEM curve
  - Enforce the DEM curve to be positive or zero
  - Assume that the DEM $\Longrightarrow$ 0 as Temp $\Longrightarrow$ 0
  - Assume that the DEM $\Longrightarrow$ 0 as Temp $\Longrightarrow$ log 10 K
- Use a minimization engine (such as AMOEBA) to solve for Fourier coefficients that describe a DEM curve that yields the best reduced χ^2 fit for observed vs. theoretical intensities.
- Apply Error Bars!!!

Definition of Error Bars

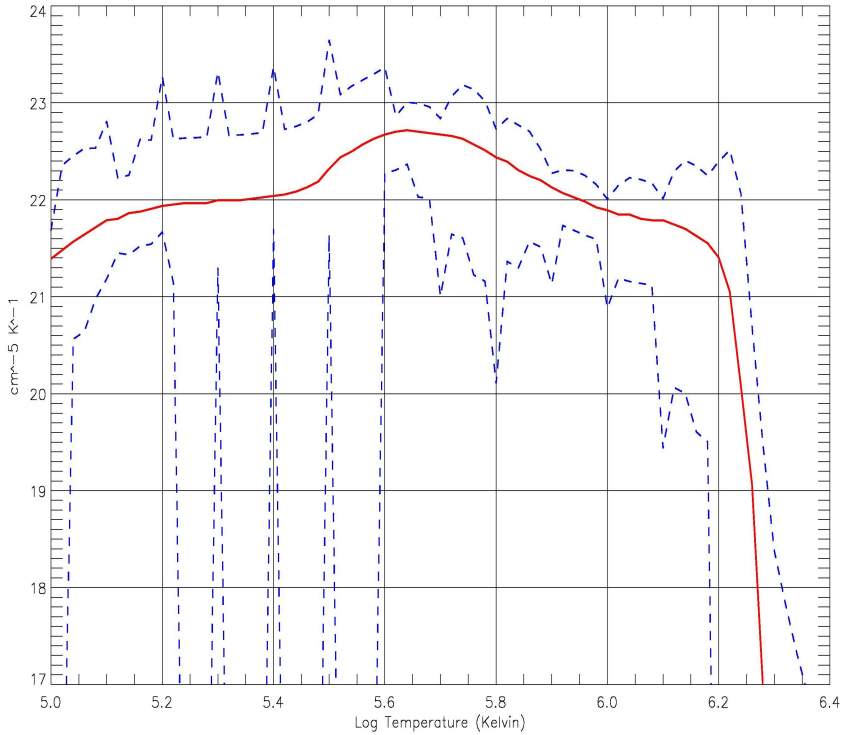
- Most published DEM curves do not have error bars despite the fact that these solutions have a tendency to be numerically unstable
- Simple definition of an error bar:
  - Vary the EM in a temperature bin by δ.
  - Fold this new DEM curve through the response functions.
  - Compare the reduced χ^2 of this solution to previous.
  - If the reduced χ^2 changes by one or greater then stop.

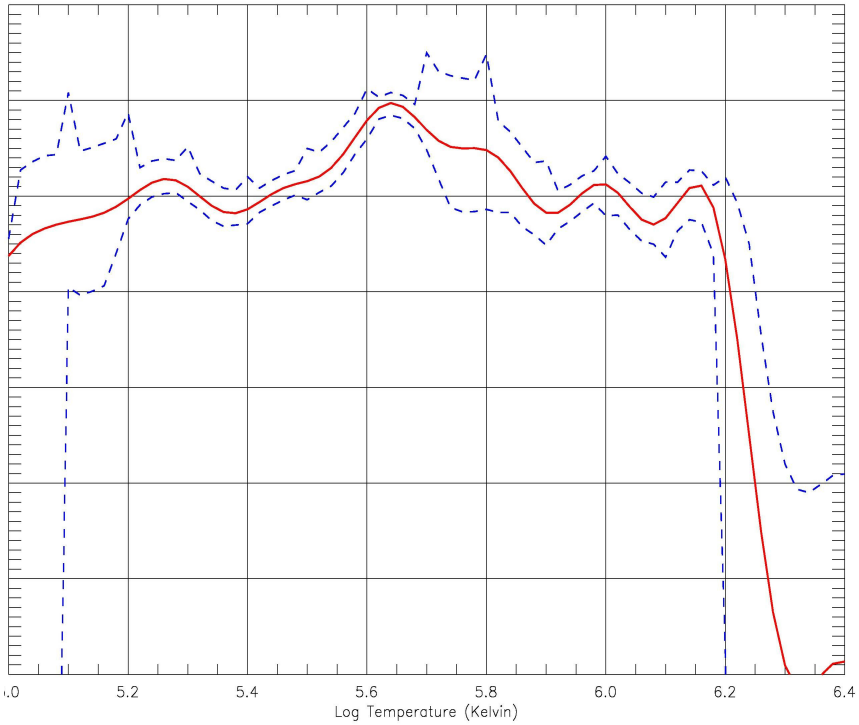
Can This Method Match an Input DEM

- Fake DEM curve (solid, red) was folded through spectral line functions to produce theoretical intensities.
- The SEMAST Fourier DEM solver then solved for the DEM (dashed, blue)



SEMAST vs. Forward Modeling





Forward Modeling

$\chi^2 = 89.7876$

SEMAST

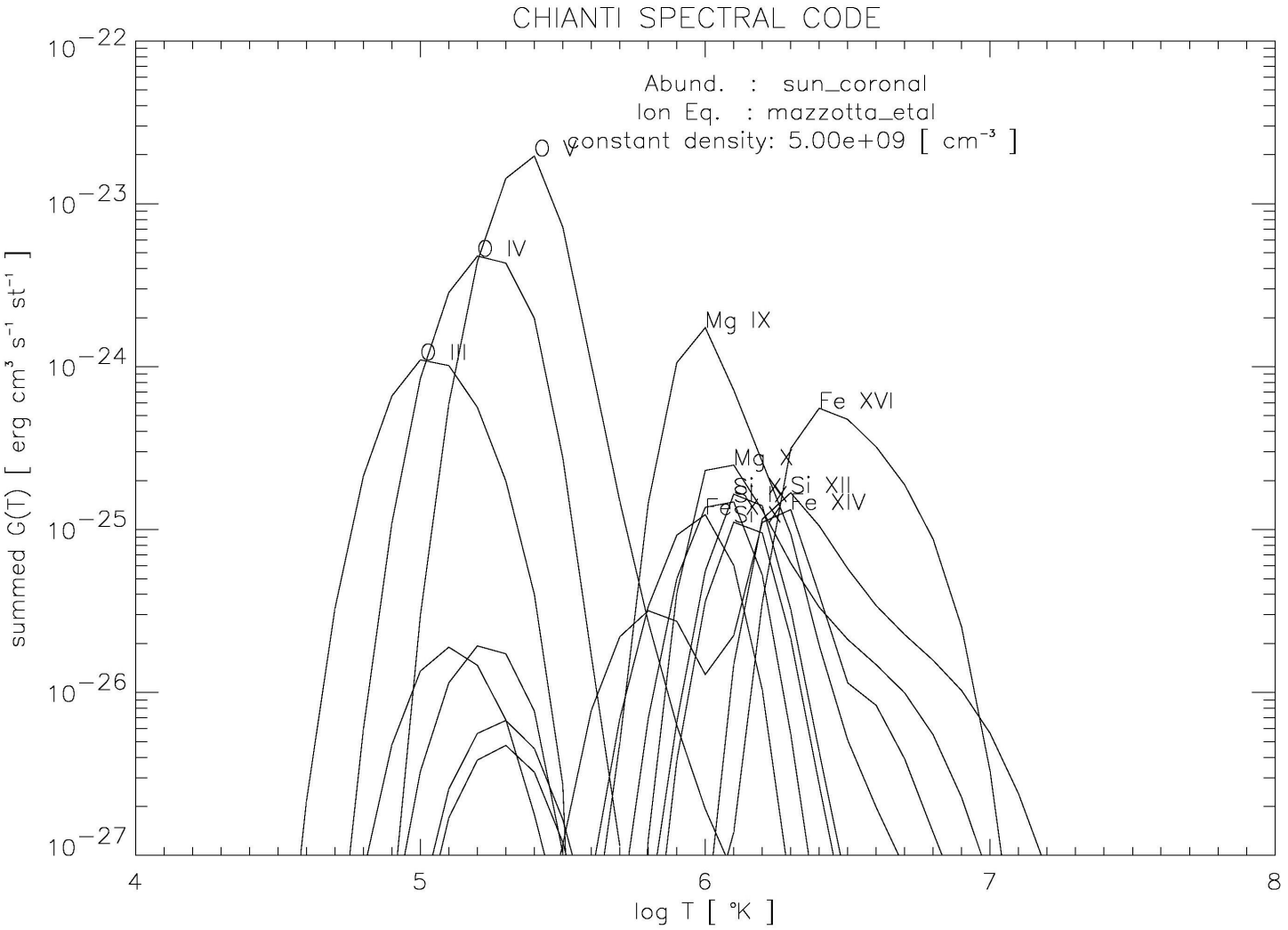
$\chi^2 = 15.1014$

18 September 2001 Loop Analysis

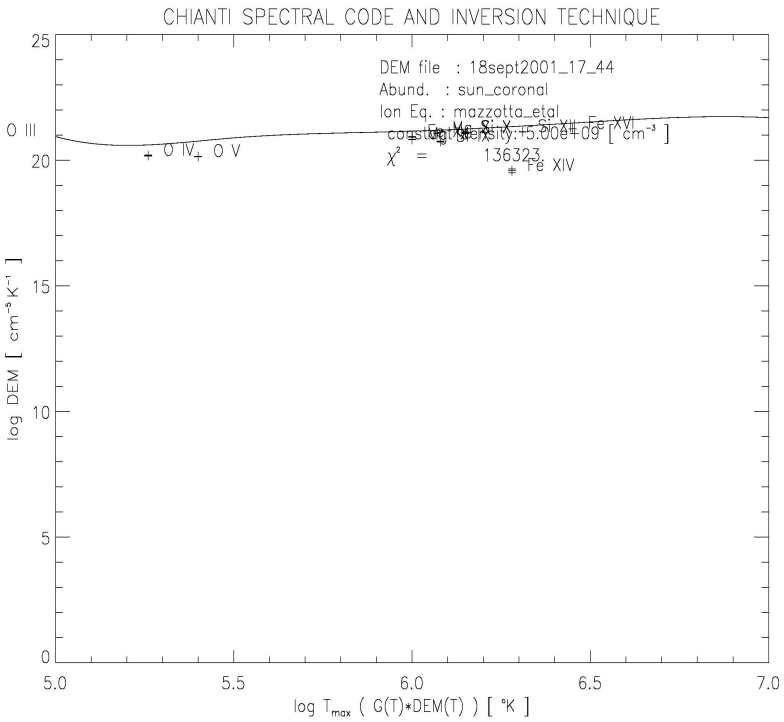
CDS lines used

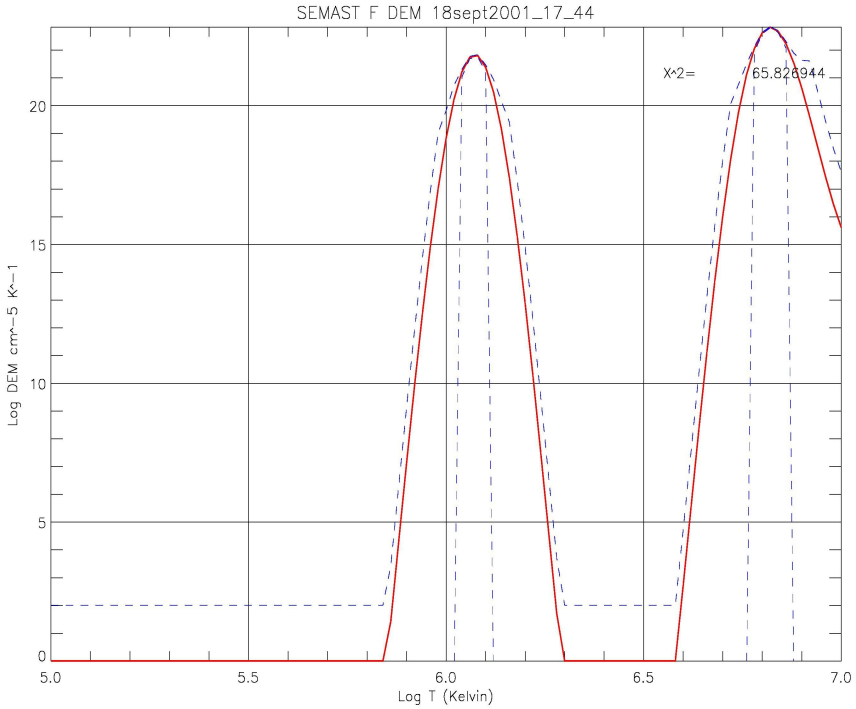
Ion	λ
O III	599.597
O IV	554.513
O V	629.732
Fe X	345.723
Mg IX	368.070
Mg X	624.941
Si IX	345.124
Si X	347.403
Si X	356.012
Fe XIV	353.831
Si XII	520.665
Fe XVI	360.761

18 September 2001 Loop Analysis
G of T Functions



SEMAST vs. CHIANTI





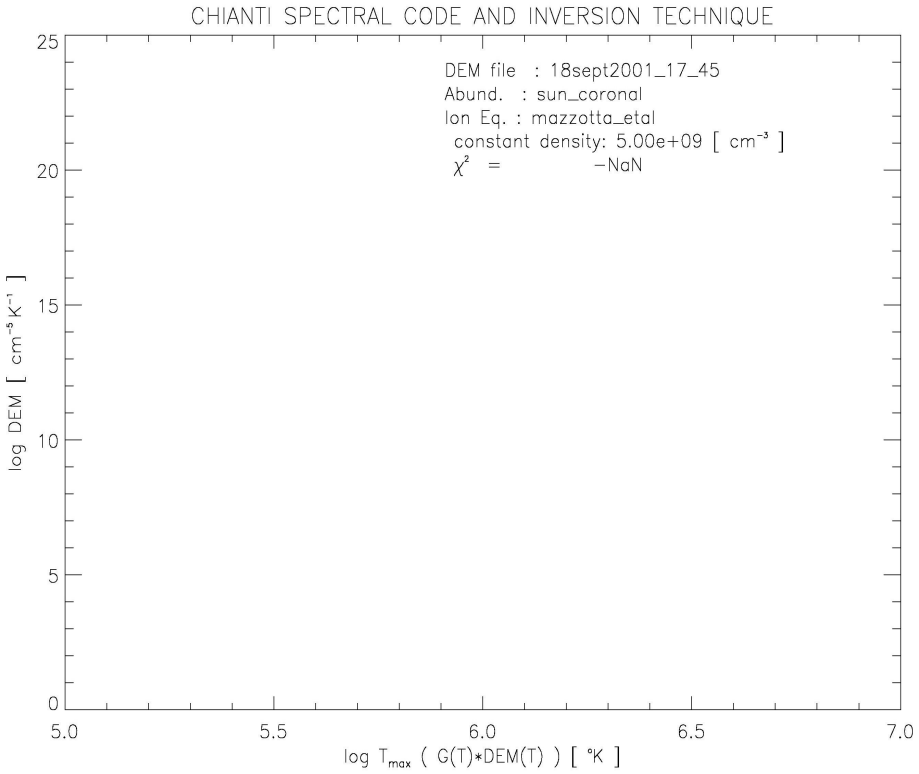
CHIANTI

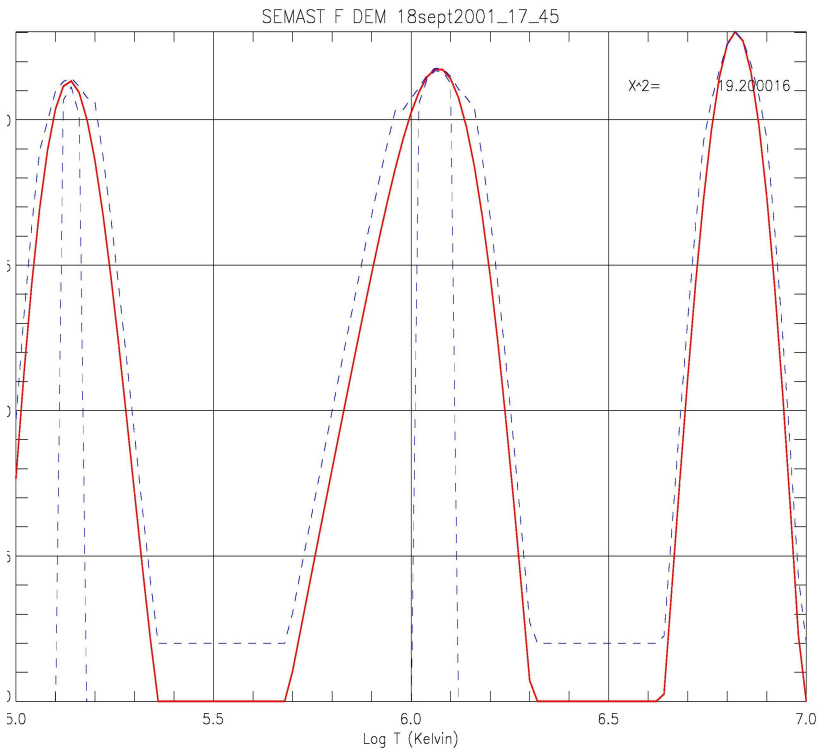
$\chi^2 = 136323$

SEMAST

$\chi^2 = 65.8270$

SEMAST vs. CHIANTI





CHIANTI

χ^2 = -NaN

SEMAST

χ^2 = 19.2000

Conclusions

- Error bars are needed on any computation of DEM
- SEMAST Fourier DEM Solver is an effective tool in the calculation of DEM curves which eliminates what may be artificial smoothness constraints.
- By proper background subtraction and eliminating biases on DEM curve shape, the initial results indicate that the 18_Sep_2001 loop analyzed is composed of multiple isothermal strands. This result would not have been possible with either Filter Ratio or standard DEM methods