

Impedance

Ph 261 Lab 10

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1 Purpose

In this lab, you will measure the magnitude of impedance as a function of frequency for a capacitor, a resistor, and an inductor.

2 Prelab

Impedance is a generalization of resistance. Impedance produces a voltage in proportion to and opposing the current, and has units of Ohms (Ω). However, impedance in capacitors and inductors differs from resistance: the current and voltage do not vary in unison (as we saw in lecture, they are $\pm\frac{\pi}{2}$ “out of phase”). For this reason, impedance is represented by a complex number. Moreover, impedance in capacitors and inductors depends on frequency. Thus, it is only well-defined when the driving (forcing) voltage or current is sinusoidal. The impedance of the three linear, passive circuit elements is:

$$\begin{array}{lll} \text{Resistor :} & Z_R = R & |Z_R| = R \\ \text{Capacitor :} & Z_C = \frac{-j}{\omega C} & |Z_C| = \frac{1}{\omega C} \\ \text{Inductor :} & Z_L = j\omega L & |Z_L| = \omega L \end{array}$$

where $j = \sqrt{-1}$.

Impedances are combined just like resistance. It is important to remember, though, that the imaginary components (the capacitors and inductors) add independently of the real components (resistors). For example, if we have a $100\ \Omega$ resistor in series with a $0.1\ \mu\text{F}$ capacitor at $\omega = 2\pi f = 2\pi\ \text{kHz}$, the total impedance is

$$\begin{aligned} Z_{tot} &= R + \frac{-j}{\omega C} \\ &= 100\ \Omega - \frac{j}{(6.28 \times 10^3)(0.1 \times 10^{-6})}\ \Omega \\ &= (100 - 1592j)\ \Omega. \end{aligned}$$

The magnitude of the impedance is then

$$\begin{aligned} |Z_{tot}| &= \sqrt{100^2 + 1592^2}\ \Omega \\ &= 1596\ \Omega. \end{aligned}$$

Exercise. Compute the magnitude of the total impedance for a $1.0\ \text{mH}$ inductor with a parasitic resistance of $30\ \Omega$ at a frequency $f = 500\ \text{kHz}$. Since inductors typically are coils comprised of many turns of thin wire, it is not uncommon to have such a parasitic resistance. The schematic of such a component is given in figure 1.

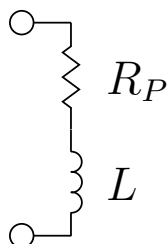


Figure 1: An inductor, L , with parasitic resistance R_P .

3 Equipment Required

- 1 Digital Multimeter
- 1 RCL Meter (shared)
- 1 Function Generator
- 1 Oscilloscope
- 1 Frequency Counter (if avail.)
- 1 Scope Probe
- 1 Protoboard
- 1 1 k Ω Resistor
- 1 0.1 μ F Capacitor
- 1 Inductor
 - various resistors for R_C .

4 Procedure

You will measure the impedance as a function of frequency for three components: a 1 k Ω resistor, 0.1 μ F capacitor, and an inductor provided to you by the TA.

1. Use the RCL meter to measure and record the value of each of the three components. Use the DMM to measure the parasitic resistance, R_P , of the inductor.
2. Breadboard the circuit drawn in figure 2 for testing each element. Use the capacitor as the first device under test (DUT). Note that

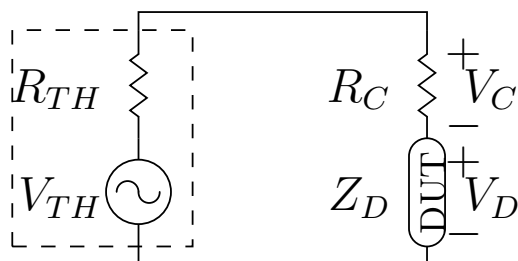


Figure 2: Setup for testing impedance.

R_{Th} is within the function generator, not a resistor on your protoboard. **Important note on scope grounding:** The grounds of the two scope probes are connected internally to the chassis, so you can't measure V_C and V_D quite as drawn. Place the probe grounds at the bottom of the DUT, and measure voltage at the top of R_C and at the top of the DUT. These are $V_C + V_D$ and V_D respectively. These two voltages will be out of phase, so subtract them using the math functions on the scope to get V_C . The voltages you want are the amplitudes of both V_C and V_D .

3. Collect the data. Please read this section carefully before you begin.
 - (a) For each device you test, draw a table in your journal, or in a spreadsheet, with room for 16 rows beneath the following 8 headings: f , V_C , V_D , R_C , i_D , $|Z_D|$ (empirical), $|Z_D|$ (theoretical), Error (%). The first 4 values are measured directly. The remaining 4 are calculated from your measurements.
 - (b) The voltages are peak-to-valley amplitudes measured from the oscilloscope.

- (c) You will collect data at 16 frequencies: from 20 Hz to 2 MHz, with three steps per decade in a 2-5-10 pattern: 20 Hz, 50 Hz, 100 Hz, 200 Hz, 500 Hz, 1 kHz, 2 kHz, ..., 2 MHz. Do not try to dial in these frequencies exactly. Just measure them with the oscilloscope or frequency counter, and record the actual values in your table.
- (d) It takes some effort to obtain reasonable accuracy with these measurements. You may change R_C as needed, but be sure you keep track of its (actual, measured) value. If V_C and V_D are both easy to measure accurately, then R_C is well-chosen.
- (e) In your table, the current through the DUT is $i_C = V_C/R_C$. The empirical $|Z_D|$ is therefore just V_D/i_C . The theoretical value should be calculated from the properties you measured initially for the DUT using the DMM and RCL meter, as well as the frequency. You may find it most convenient to set up a spreadsheet ahead of time so that these quantities and the percentage error are calculated as you go. This may help you recognize problems.
- (f) Repeat the above steps with the resistor, and then the inductor, as the DUT. When you work out the theoretical impedance values for the inductor, be sure to include the parasitic resistance. Refer to the example and exercise in the pre-lab.

5 Conclusion

1. Print out the data tables and secure them in your lab journal.
2. On one log-log graph, plot the theoretical and empirical magnitude of the impedance for the capacitor as a function of frequency, f . Do the same for the resistor and the inductor. The theoretical curves, aside from any parasitic behavior, should be straight lines as in the plot provided in this week's lecture notes.
3. You measured the parasitic resistance of the inductor. At what frequencies does this resistance become a significant part of the impedance?
4. You might also have observed a divergence of the empirical and theoretical impedances of the inductor at high frequencies. The parasitic resistance model (figure 1) explains the divergence at low frequencies. In your journal, sketch a model of an inductor that could explain the high frequency divergence.