

OUTLINE

LAST TIME: KVL, KCL, RESISTORS IN SERIES & PARALLEL

TODAY (CH. 4):

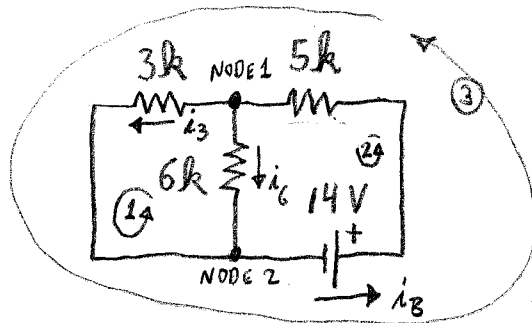
VOLTAGE DIVIDERS

CURRENT DIVIDERS (AND CONDUCTANCE,  $G \equiv 1/R$ )

VISUAL: BRING A POTENTIOMETER.

REVIEW KVL, KCL, SERIES & PARALLEL RESISTORS

FIND THE CURRENT IN THE BATTERY:



METHOD 1: KCL & KVL

\* KCL (1):  $-i_3 - i_6 + i_B = 0$

KCL (2): SAME!

\* KVL (1):  $-6k i_6 + 3k i_3 = 0$   
 $\Rightarrow i_3 = 2i_6$

\* KVL (2):  $14V - 5k i_B - 6k i_6 = 0$

KVL (3):  $14V - 5k i_B - 3k i_3 = 0$  ← REDUNDANT!

\* = 3 EQ; 3 UNKNOWN CURRENTS.

KCL (1) & KVL (1):  $i_B = 3i_6$

SUB THIS INTO KVL (2):

$$14V - 5k i_B - 2i_B = 0$$

$$\Rightarrow 7k i_B = 14V \Rightarrow i_B = 2mA$$

CONT. →

SAMPAL

METHOD 2: FIND THE EQUIVALENT RESISTANCE

3k & 6k ARE IN PARALLEL:

$$R_{36} = \frac{1}{\frac{1}{6k} + \frac{1}{3k}} = \frac{1}{\frac{3}{6k}} = 2k$$

R<sub>36</sub> IS IN SERIES WITH 5k:

$$R_{TOT} = 2k + 5k = 7k$$

$$\underline{\underline{SO}} \quad I_B = \frac{14V}{7k\Omega} = 2mA$$

SAME RESULT, MUCH FASTER!

ASIDE:

CONDUCTANCE IS DEFINED AS  $G \equiv 1/R$ .

RECALL THAT FOR RESISTORS IN SERIES,

$$R_{TOT} = \sum R_n$$

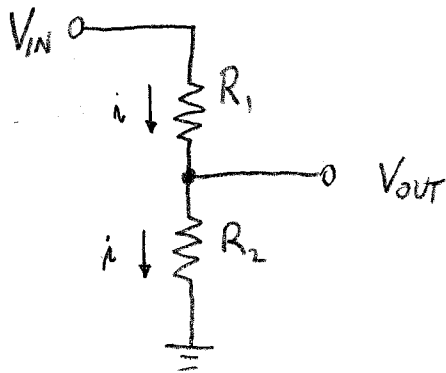
SIMILARLY, FOR RESISTORS IN PARALLEL,

$$G_{TOT} = \frac{1}{R_{TOT}} = \sum \frac{1}{R_n} = \sum G_n$$

CAMPAD

# VOLTAGE DIVIDERS

RESISTORS IN SERIES CAN BE USED TO SCALE DOWN AN INPUT VOLTAGE.



WHAT IS  $V_{OUT}$  IN TERMS OF  $V_{IN}$ ?

$$i = \frac{V_{IN}}{R_1 + R_2} \quad (\text{SERIES RESISTORS})$$

$$V_{OUT} = i R_2 \quad (\text{OHM'S LAW})$$

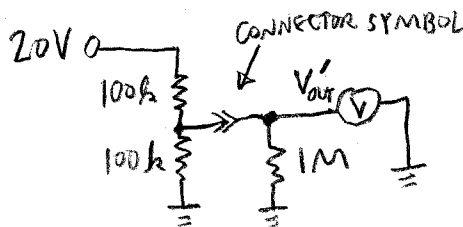
SO  $V_{OUT} = V_{IN} \left( \frac{R_2}{R_1 + R_2} \right)$

EXAMPLE LET  $R_1 = R_2 = 100k\Omega$ ,  $V_{IN} = 20V$

Q: WHAT IS  $V_{OUT}$ ?

A:  $V_{OUT} = 20V \left( \frac{100k}{200k} \right) = 10V$

NOW, SUPPOSE I MEASURE  $V_{OUT}$  USING A VOLTMETER WITH  $1M\Omega$  INPUT IMPEDANCE



Q: WHAT IS  $V'_{OUT}$ ?

A: STILL A VOLTAGE DIVIDER, BUT  $R'_2 = \frac{1}{\frac{1}{100k} + \frac{1}{1M}} = 91k$

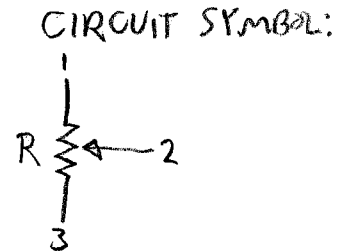
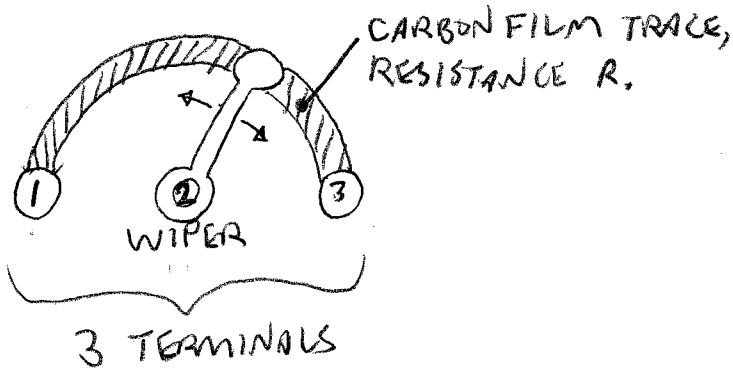
SO  $V'_{OUT} = \frac{91k}{100 + 91k} = 9.5V$

Q: HOW WOULD I GET A MORE ACCURATE RESULT?

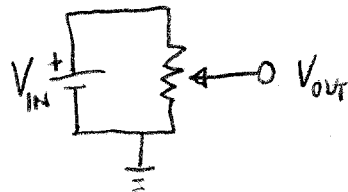
# POTENTIOMETERS

A POTENTIOMETER (ALSO CALLED A POT, RHEOSTAT, OR VARIABLE RESISTOR) PROVIDES AN ADJUSTABLE RESISTANCE.

TYPICAL ROTARY POT:

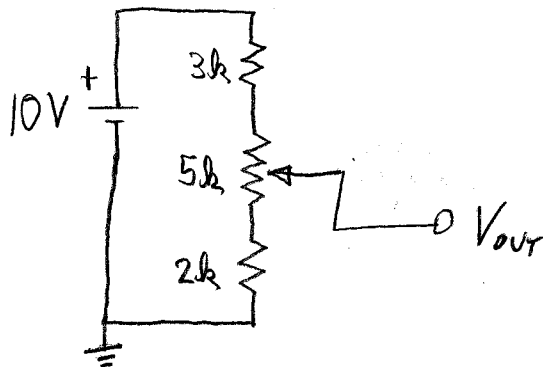


A POT IS USEFUL AS A VOLTAGE DIVIDER:



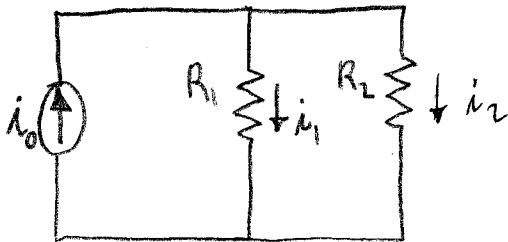
ADJUSTABLE OUTPUT,  $0 \leq V_{OUT} \leq V_{IN}$

Q: WHAT IS THE OUTPUT VOLTAGE RANGE OF THIS CIRCUIT?



# CURRENT DIVISION

HOW IS CURRENT DISTRIBUTED AMONG A SET OF RESISTORS IN PARALLEL?



$$\text{KCL: } i_0 = i_1 + i_2$$

$$\Rightarrow i_2 = i_0 - i_1$$

SINCE THESE RESISTORS ARE IN PARALLEL, THEY MUST HAVE THE SAME VOLTAGE DROP. THIS CAN BE SHOWN

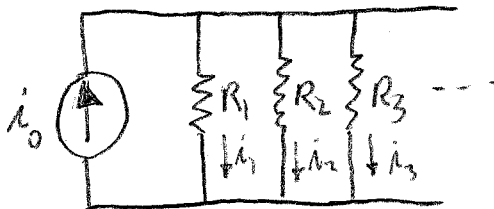
$$\text{WITH KVL: } i_1 R_1 - i_2 R_2 = 0$$

$$i_1 R_1 - (i_0 - i_1) R_2 = 0$$

$$\Rightarrow i_1 (R_1 + R_2) = i_0 R_2$$

$$i_1 = i_0 \frac{R_2}{R_1 + R_2}; \quad \text{SIMILARLY, } i_2 = i_0 \frac{R_1}{R_1 + R_2}$$

HOW DOES THIS GENERALIZE TO A LARGER NUMBER OF RESISTORS?



$$\text{KCL: } i_0 = \sum_{\ell=1}^N i_{\ell}$$

$$\text{KVL: } i_1 R_1 = i_2 R_2 = \dots = i_N R_N$$

$$\Rightarrow i_{\ell} = i_m \frac{R_m}{R_{\ell}} \quad \text{FOR ANY } \ell, m$$

SUBSTITUTING KVL INTO KCL,

$$i_0 = \sum_{\ell} i_m \frac{R_m}{R_{\ell}} = i_m R_m \sum_{\ell} \frac{1}{R_{\ell}} \Rightarrow i_m = \frac{i_0}{R_m \sum_{\ell} \frac{1}{R_{\ell}}}$$

THIS IS COMMONLY WRITTEN

$$i_m = \frac{i G_m}{\sum_{\ell} G_{\ell}}, \quad \text{WHERE } G_{\ell} \equiv \frac{1}{R_{\ell}} \text{ IS THE CONDUCTIVITY OF } \ell\text{TH RESISTOR.}$$

## NODE VOLTAGE ANALYSIS

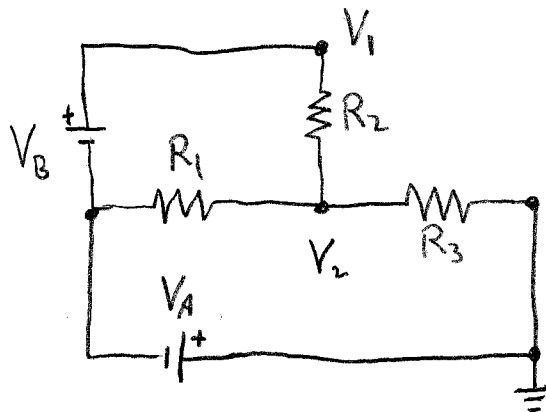
NOTE — THIS IS NOT IN YOUR TEXT!

"NODE VOLTAGE ANALYSIS" IS A SYSTEMATIC APPROACH TO CIRCUIT ANALYSIS.

THE FOLLOWING PROCEDURE GIVES  $N$  EQUATIONS FOR  $N$  UNKNOWN VOLTAGES:

- ① LABEL THE UNKNOWN VOLTAGES ( $V_1, V_2, \dots$ ) AT EACH NODE.
- ② CHOOSE A REFERENCE VOLTAGE (GROUND, IF NOT ALREADY SPECIFIED).
- ③ EXPRESS  $V$  AT EACH NODE IN TERMS OF
  - Ⓐ KNOWN QUANTITIES (AS FAR AS POSSIBLE) OR
  - Ⓑ OTHER UNKNOWN VOLTAGES (USING KCL AND OHM'S LAW AS NEEDED).
- ④ SOLVE THE RESULTING SYSTEM OF EQUATIONS.

EXAMPLE



GIVEN:  $R_1, R_2, R_3$

FIND:  $V_1, V_2$

STEPS 1 & 2 ARE ALREADY DONE IN THE SCHEMATIC.

$$(3a) \quad V_1 = -V_A + V_B$$

(3b) USING KCL AT NODE 2,

$$\frac{V_1 - V_2}{R_2} + \frac{-V_A - V_2}{R_1} + \frac{0 - V_2}{R_3} = 0$$

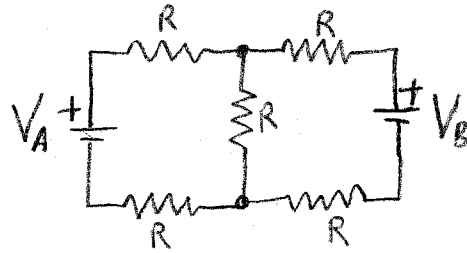
$$\Rightarrow \frac{-V_A + V_B - V_2}{R_2} - \frac{V_A + V_2}{R_1} - \frac{V_2}{R_3} = 0$$

$$\Rightarrow V_2 = \frac{\frac{V_B - V_A}{R_2} - \frac{V_A}{R_1}}{1/R_1 + 1/R_2 + 1/R_3}$$

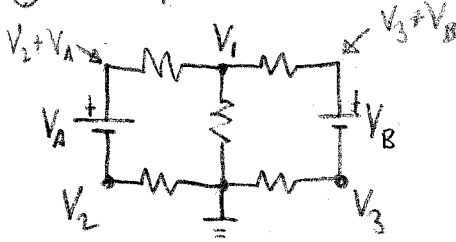
EXAMPLE — HANDLING VOLTAGE SOURCES GRACEFULLY

THE EXISTENCE OF MULTIPLE VOLTAGE SOURCES CAN MAKE NODE VOLTAGE ANALYSIS A LITTLE TRICKIER.

(ASSUME  $V_A, V_B, R$  KNOWN)



- ① LABEL NODES;
- ② PICK GROUND



STRATEGY: I HAVE NAMED ONLY 1 NODE ADJACENT TO EACH VOLTAGE SOURCE.

KCL EQUATIONS FOR EACH NODE:

NODE 1

$$\frac{(V_2 + V_A) - V_1}{R} + \frac{0 - V_1}{R} + \frac{(V_3 + V_B) - V_1}{R} = 0$$

$$\Rightarrow 3V_1 = V_2 + V_3 + V_A + V_B$$

NODE 2

$$\frac{V_1 - (V_2 + V_A)}{R} + \frac{0 - V_2}{R} = 0 \Rightarrow V_2 = \frac{(V_1 + V_A)}{2}$$

NODE 3

$$\frac{V_1 - (V_3 + V_B)}{R} + \frac{0 - V_3}{R} = 0 \Rightarrow V_3 = \frac{(V_1 + V_B)}{2}$$

PLUGGING IN SOLUTIONS FOR  $V_2, V_3$  INTO NODE 1:

$$3V_1 = \frac{1}{2}(V_1 + V_A) + \frac{1}{2}(V_1 + V_B) + V_A + V_B$$

$$\Rightarrow 6V_1 = 2V_1 + 3V_A + 3V_B \Rightarrow V_1 = \frac{3}{4}(V_A + V_B)$$

WHICH LEADS TO

$$V_2 = \frac{1}{2} \left[ \frac{3}{4}(V_A + V_B) + V_A \right] = \frac{7}{8}V_A + \frac{3}{8}V_B$$

$$V_3 = \frac{3}{8}V_A + \frac{7}{8}V_B$$

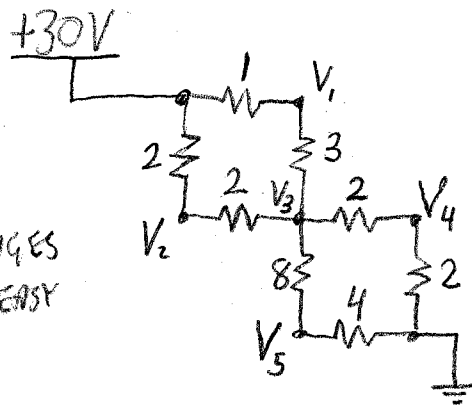


### EXAMPLE WITH MORE NODES

(RESISTANCES IN OHMS)

FIND  $V_1, V_2, V_3, V_4, V_5$ .

NOTE THAT ONCE ALL THE VOLTAGES ARE KNOWN, CURRENTS ARE EASY TO FIND IF YOU WANT THEM!



EQUATIONS FOR EACH NODE USING KCL:

$$\textcircled{1} \frac{30-V_1}{1} + \frac{V_3-V_1}{3} = 0 \Rightarrow V_1 = \frac{90+V_3}{4}$$

$$\textcircled{2} \frac{V_3-V_2}{2} + \frac{30-V_2}{2} = 0 \Rightarrow V_2 = 15 + \frac{1}{2}V_3$$

$$\textcircled{3} \frac{V_1-V_3}{3} + \frac{V_2-V_3}{2} + \frac{V_5-V_3}{8} + \frac{V_4-V_3}{2} = 0$$

$$\textcircled{4} \frac{V_2-V_4}{2} + \frac{0-V_4}{8} = 0 \Rightarrow V_4 = \frac{1}{2}V_3$$

$$\textcircled{5} \frac{0-V_3}{4} + \frac{V_3-V_5}{8} = 0 \Rightarrow V_5 = \frac{1}{3}V_3$$

5 EQ'S  
5 UNKNOWN'S

\* $\textcircled{3}$  CAN BE WRITTEN:

$$8V_1 - 8V_2 + 12V_2 - 12V_3 + 3V_5 - 3V_3 + 12V_4 - 12V_3 = 0$$

$$\Rightarrow 35V_3 = 8V_1 + 12V_2 + 3V_5 + 12V_4$$

NOW, PLUG IN SOLN'S FOR  $V_1, V_2, V_4, V_5$  IN TERMS OF  $V_3$ :

$$35V_3 = (180 + 2V_3) + (180 + 6V_3) + V_3 + 6V_3 = 360 + 15V_3$$

$$\Rightarrow 20V_3 = 360 \Rightarrow V_3 = 18V$$

PUT THIS BACK INTO THE SOLN'S FOR  $V_1, V_2, V_4, V_5$ :

$$V_1 = \frac{90+18}{4} = 27V$$

$$V_2 = 15+9 = 24V$$

$$V_4 = 9V$$

$$V_5 = 6V$$