

OUTLINE

LAST TIME:

VOLTAGE DIVIDERS

CURRENT DIVIDERS

NODE VOLTAGE ANALYSIS

TODAY:

MORE EXAMPLES (CURRENT & VOLTAGE DIVIDERS, NVA)  
THEVENIN AND NORTON EQUIVALENTS

SHOW &amp; TELL:

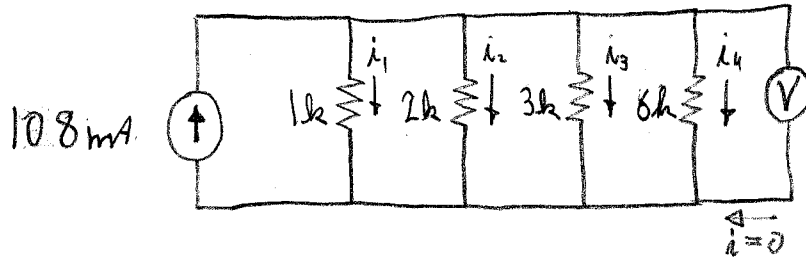
WHEATSTONE BRIDGE APPARATUS (IF AVAILABLE)

EXAM NEXT WEEK —

COVERS LECTURES 1-4 (CHS. 2, 3, 4, 6)

4 PROBLEMS: 1 CORRESPONDING TO EACH LECTURE

EXAMPLE — CURRENT DIVIDER



- WHAT IS  $V$ ?
- FIND THE CURRENTS IN ALL 4 RESISTORS.

THE GENERAL FORMULA IS  $i_R = \frac{G_R}{G_{TOT}} i_{TOT}$ .

SUMMING PARALLEL CONDUCTANCES,

$$G_{TOT} = \left( \frac{1}{1k} + \frac{1}{2k} + \frac{1}{3k} + \frac{1}{6k} \right) \Omega^{-1}$$

$$= \frac{12+6+4+2}{12k} \Omega^{-1} = 2 \times 10^{-3}$$

SO,

$$i_1 = \frac{G_1}{G_{TOT}} \cdot 108 \text{ mA} = \frac{1/1k}{2 \times 10^{-3}} \cdot 108 \text{ mA} = \frac{108}{2} \text{ mA} = 54 \text{ mA}$$

SIMILARLY,

(SKIP)  $i_2 = \frac{1/2k}{2 \times 10^{-3}} \cdot 108 \text{ mA} = \frac{108}{4} \text{ mA} = 27 \text{ mA}$

$$i_3 = \frac{1/3k}{2 \times 10^{-3}} \cdot 108 \text{ mA} = \frac{108}{6} \text{ mA} = 18 \text{ mA}$$

(SKIP)  $i_4 = \frac{1/6k}{2 \times 10^{-3}} \cdot 108 \text{ mA} = \frac{108}{12} \text{ mA} = 9 \text{ mA}$

AND  $V = iR$

E.G.  $V = i_1 \times 1k\Omega = 54 \text{ V}$ ,

OR  $V = i_3 \times 3k\Omega = 54 \text{ V}$  ✓

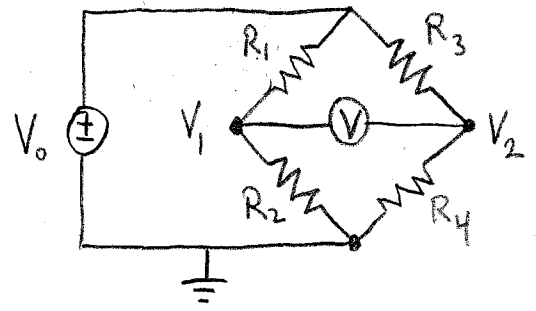
ETC.

OF COURSE THEY ALL MUST AGREE!

# VOLTAGE DIVIDER EXAMPLE — THE WHEATSTONE BRIDGE (A CLASSIC INSTRUMENT OF THE 19<sup>TH</sup> CENTURY)

THIS IS A CIRCUIT USED TO COMPARE RESISTORS.

Q: UNDER WHAT CONDITIONS DOES THE VOLTMETER READ ZERO?



A: WHEN  $V_1 = V_2$ . SINCE THERE IS NO CURRENT WHEN THE VOLTMETER READS  $V=0$ , WE CAN ANALYZE THE LEFT & RIGHT SIDES AS COMPLETELY INDEPENDENT VOLTAGE DIVIDERS:

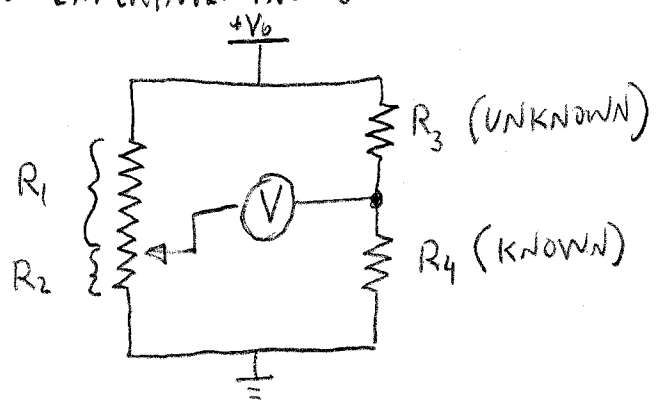
$$V_1 = V_0 \frac{R_2}{R_1 + R_2}$$

$$V_2 = V_0 \frac{R_4}{R_3 + R_4}$$

$$\text{SO } V_1 = V_2 \iff \frac{R_2}{R_1 + R_2} = \frac{R_4}{R_3 + R_4} \iff \frac{R_1 + R_2}{R_2} = \frac{R_3 + R_4}{R_4}$$

$$\iff 1 + \frac{R_1}{R_2} = 1 + \frac{R_3}{R_4} \iff \boxed{\frac{R_1}{R_2} = \frac{R_3}{R_4}}$$

TYPICAL EXPERIMENTAL SETUP:



$R_1$  &  $R_2$  IS A POTENTIOMETER MADE OF A LONG, THIN, VERY UNIFORM WIRE. THE RATIO  $R_1/R_2$  IS A MEASURED RATIO OF LENGTHS ABOVE & BELOW THE WIPER.

BECAUSE THE CURRENT IN THE VOLTMETER IS EXACTLY ZERO WHEN  $V=0$ , THE VOLTMETER REQUIRES NO CALIBRATION. IF  $R_4$  IS ACCURATELY KNOWN, THEN WE CAN MEASURE  $R_3$  VERY WELL:

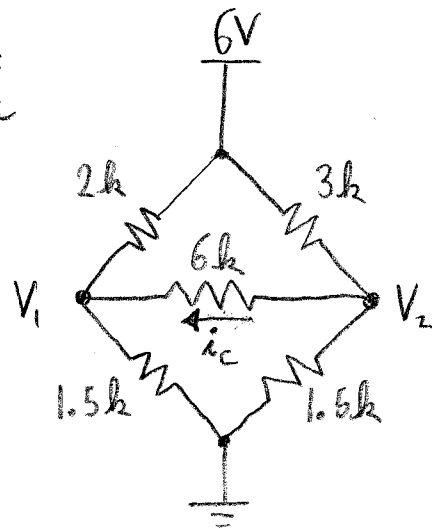
$$R_3 = R_4 \left( \frac{R_1}{R_2} \right)$$

IT IS NOT TOO HARD TO GET 3-4 SIGNIFICANT FIGURES THIS WAY.

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## NVA EXAMPLE — BRIDGE CIRCUIT

THIS IS AN UNBALANCED BRIDGE, SO THERE SHOULD BE CURRENT THROUGH THE CENTRAL RESISTOR. FIND  $i_c$ .



NODE 1:

$$\frac{6V - V_1}{2k} + \frac{0 - V_1}{1.5k} + \frac{V_2 - V_1}{6k} = 0$$

$$\Rightarrow 18V - 3V_1 - 4V_1 + V_2 - V_1 = 0$$

$$\Rightarrow V_2 = 8V_1 - 18V$$

NODE 2:

$$\frac{6V - V_2}{3k} + \frac{0 - V_2}{1.5k} + \frac{V_1 - V_2}{6k} = 0$$

$$\Rightarrow 12V - 2V_2 - 4V_2 + V_1 - V_2 = 0$$

$$\Rightarrow V_1 = 7V_2 - 12V$$

$$= 7(8V_1 - 18V) - 12V$$

$$\Rightarrow V_1 = 56V_1 - 126V - 12V \Rightarrow 55V_1 = 138V$$

$$\therefore V_1 = 2.51V$$

$$\text{SO } V_2 = 8(2.51V) - 18V = 2.08V$$

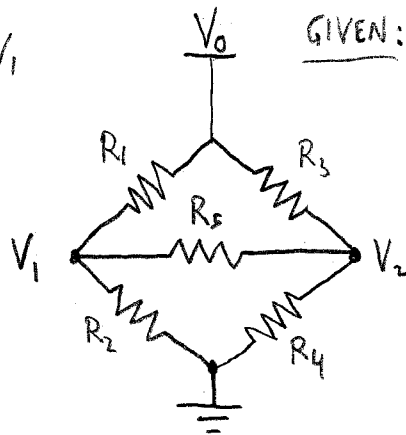
SO THE CURRENT IS

$$i_c = \frac{V_2 - V_1}{6k} = \frac{-0.43V}{6k} = -0.0717 \text{ mA}$$

$$i_c = -71.7 \mu\text{A}$$

NVA EXAMPLE : FIND  $V_1, V_2, V_2 - V_1$   
 (COMPARE FAISLER, CH. 5)

GIVEN:  $V_0, R_2$



NODE 1

$$\frac{V_0 - V_1}{R_1} + \frac{V_2 - V_1}{R_5} + \frac{0 - V_1}{R_2} = 0$$

$$\Rightarrow V_1 \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_5} \right) = \frac{V_0}{R_1} + \frac{V_2}{R_5} \quad (0)$$

$$\Rightarrow V_1 = \frac{G_1 V_0 + G_5 V_2}{G_1 + G_2 + G_5} \quad (1)$$

NODE 2

$$\frac{V_0 - V_2}{R_3} + \frac{V_1 - V_2}{R_5} + \frac{0 - V_2}{R_4} = 0 \Rightarrow V_2 (G_3 + G_4 + G_5) = G_3 V_0 + G_5 V_1 \quad (2)$$

PLUGGING (1) INTO (2),

$$V_2 (G_3 + G_4 + G_5) = G_3 V_0 + \frac{G_1 G_5 V_0 + G_5^2 V_2}{G_1 + G_2 + G_5}$$

$$\Rightarrow V_2 (G_3 + G_4 + G_5) (G_1 + G_2 + G_5) - V_2 G_5^2 = (G_1 + G_2 + G_5) G_3 V_0 + G_1 G_5 V_0$$

$$\Rightarrow V_2 = \frac{(G_1 G_3 + G_2 G_3 + G_3 G_5 + G_1 G_5) V_0}{G_1 G_3 + G_1 G_4 + G_1 G_5 + G_2 G_3 + G_2 G_4 + G_2 G_5 + G_3 G_5 + G_4 G_5}$$

SIMILARLY,

$$V_1 = \frac{(G_1 G_3 + G_1 G_4 + G_1 G_5 + G_3 G_5) V_0}{G_1 G_3 + G_1 G_4 + G_1 G_5 + G_2 G_3 + G_2 G_4 + G_2 G_5 + G_3 G_5 + G_4 G_5}$$

WE ARE ALSO INTERESTED IN  $V_2 - V_1$ :

$$V_2 - V_1 = \frac{G_2 G_3 - G_1 G_4}{G_1 G_3 + G_1 G_4 + G_1 G_5 + G_2 G_3 + G_2 G_4 + G_2 G_5 + G_3 G_5 + G_4 G_5}$$

NOTE THAT  $V_2 = V_1$  IFF  $G_2 G_3 = G_1 G_4$  ← ALL 3 MATHEMATICALLY EQUIVALENT.

$$\text{SO } V_2 = V_1 \Leftrightarrow \frac{R_1}{R_2} = \frac{R_3}{R_4} \Leftrightarrow \frac{R_2}{R_1 + R_2} = \frac{R_4}{R_3 + R_4}$$

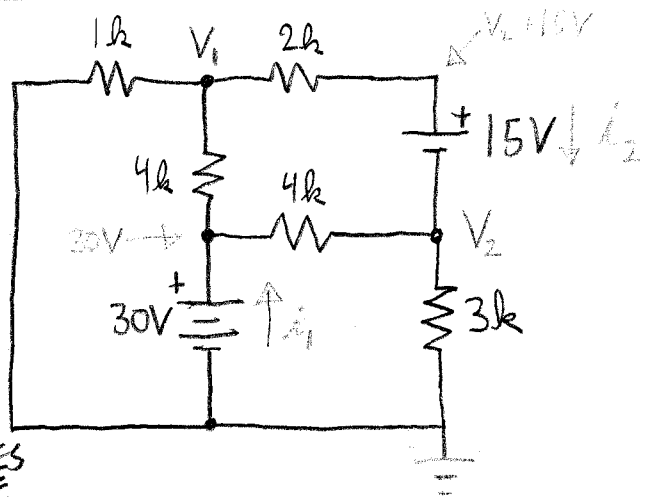
DOES THIS MAKE SENSE? THINK OF VOLTAGE DIVIDERS.

## NVA EXAMPLE

FIND THE TOTAL POWER DISSIPATED.

SOLVING STRATEGY:

- NAME AS FEW NODES AS POSSIBLE FOR SIMPLICITY. IN PARTICULAR, VOLTAGE SOURCES DO NOT NEED NODES AT BOTH ENDS.
- CALCULATE POWER AT THE SOURCES (IV AT BATTERIES). THERE ARE FEWER BATTERIES THAN RESISTORS!



$$\text{NODE 1: } \frac{30V - V_1}{4k} + \frac{0 - V_1}{1k} + \frac{(V_2 + 15V) - V_1}{2k} = 0$$

$$\Rightarrow V_1 = \frac{60V + 2V_2}{7} \quad (\text{SKIPPED SOME ALGEBRA!})$$

$$\text{NODE 2: } \frac{V_1 - (V_2 + 15V)}{2k} + \frac{30V - V_2}{4k} + \frac{0 - V_2}{3k} = 0$$

$$\Rightarrow 13V_2 = 6V_1 + 180V$$

NOW, SUBSTITUTE THE NODE 1 SOLN. AND ISOLATE  $V_2$ :

$$79V_2 = 1620V \Rightarrow V_2 = 20.5V$$

$$\Rightarrow V_1 = \frac{60V + 2(20.5V)}{7} = 14.4V$$

NEXT FIND CURRENTS THROUGH THE BATTERIES:

$$\text{KVL: } i_1 = \frac{V_1 - 0}{1k} + \frac{V_2 - 0}{3k} = 14.4\text{mA} + 6.8\text{mA} = 21.2\text{mA}$$

$$i_2 = \frac{V_1 - (V_2 + 15V)}{2k} = 10.6\text{mA}$$

POWER:

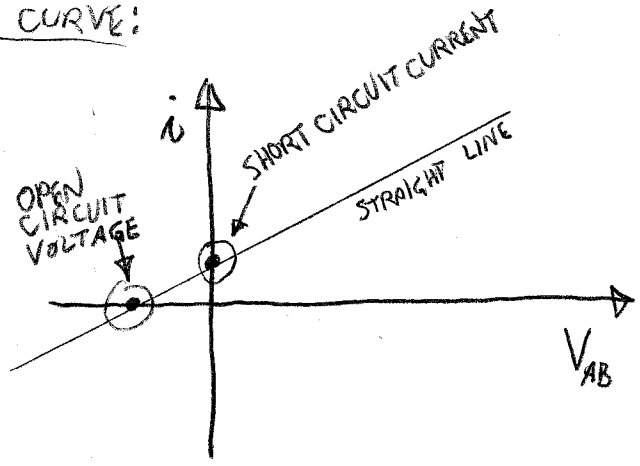
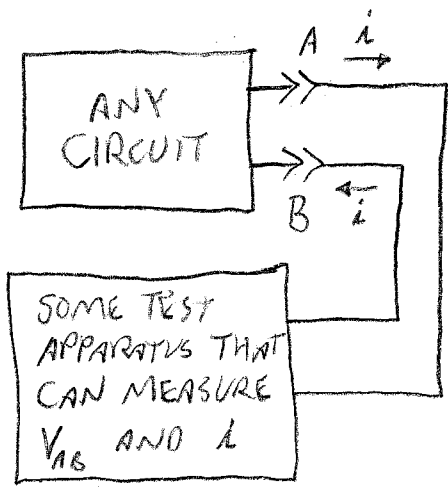
$$P = 30V \cdot i_1 + 15V \cdot i_2 = 795\text{mW} \quad (\approx 0.8\text{W})$$

# THEVENIN'S THEOREM \*

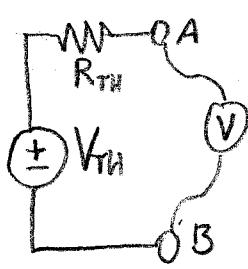
"ANY NETWORK OF RESISTORS, VOLTAGE SOURCES AND CURRENT SOURCES WITH TWO TERMINALS IS INDISTINGUISHABLE ELECTRICALLY FROM A SINGLE VOLTAGE SOURCE IN SERIES WITH A RESISTOR."

WE WILL NOT PROVE THIS THEOREM, BUT I WILL SHOW YOU HOW TO FIND THE THEVENIN EQUIVALENT OF ANY CIRCUIT.

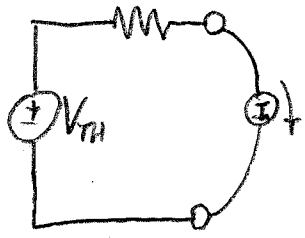
IN CONCEPT, IT WORKS LIKE THIS. THE CIRCUIT CAN BE CHARACTERIZED BY ITS IV CURVE:



THEVENIN'S THEOREM WORKS BECAUSE ANY COMBINATION OF RESISTORS, VOLTAGES AND CURRENT SOURCES HAS A STRAIGHT LINE IV CURVE. THE THEVENIN EQUIVALENT IS THEN:



OPEN CIRCUIT VOLTAGE  
 $V_{OC} = V_{TH}$



SHORT CIRCUIT CURRENT  
 $I_{SC} = \frac{V_{TH}}{R_{TH}}$

SO

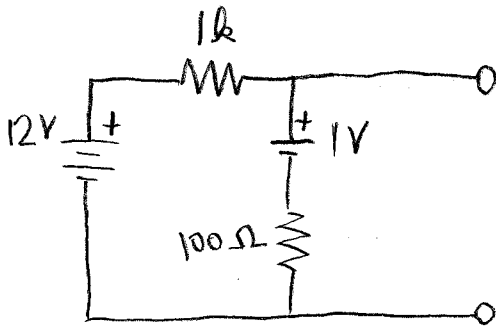
$$V_{TH} = V_{OC}$$

$$R_{TH} = \frac{V_{TH}}{I_{SC}}$$

\* THE WIKIPEDIA ENTRY ON THIS TOPIC IS VERY GOOD.

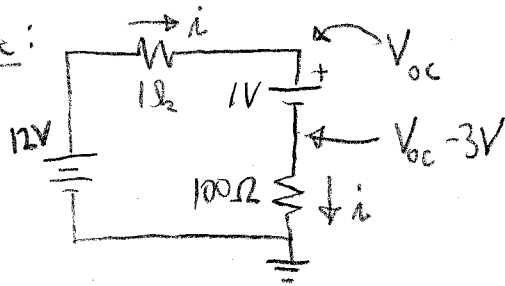
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EXAMPLE



FIND THE THÉVENIN EQUIVALENT.

FIND  $V_{oc}$ :



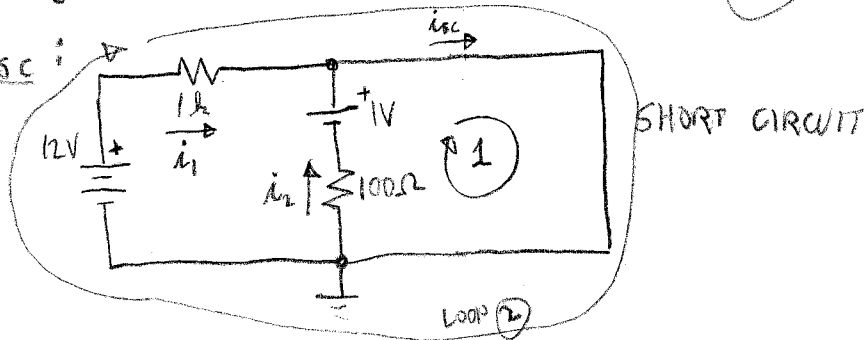
$$KVL: 12V - i \cdot 1k\Omega - 1V - i \cdot 100\Omega = 0$$

$$i \cdot 1.1k\Omega = 11V$$

$$i = \frac{11V}{1.1k\Omega} = 10 \text{ mA}$$

$$V_{oc} = 100\Omega \cdot i + 3V = 1V + 3V = 4V$$

FIND  $i_{sc}$ :

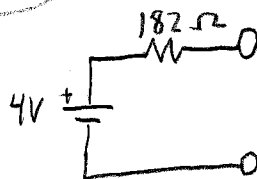


$$KVL \textcircled{1}: -i_2 \cdot 100\Omega + 1V = 0 \Rightarrow i_2 = 10 \text{ mA}$$

$$KVL \textcircled{2}: 12V - i_1 \cdot 1k = 0 \Rightarrow i_1 = 12 \text{ mA}$$

$$KCL: i_{sc} = i_1 + i_2 = 22 \text{ mA} \Rightarrow R_{Th} = \frac{4V}{22 \text{ mA}} = 182\Omega$$

THÉVENIN EQUIVALENT:

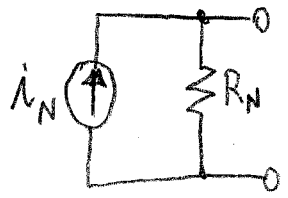




# NORTON EQUIVALENT

FOR EVERY THÉVENIN EQUIVALENT, THERE IS A NORTON EQUIVALENT (AND VICE VERSA), PROVIDED  $V_{oc}$  AND  $i_{sc}$  ARE FINITE.

THE NORTON EQUIVALENT TAKES THIS FORM:

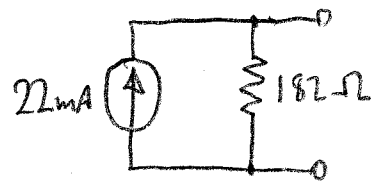


WHERE  $i_N = i_{sc}$ , AND  
 $V_{oc} = i_N R_N$ .

Q: WHAT IS THE THE NORTON EQUIVALENT FOR THE PREVIOUS EXAMPLE?

A: WE HAD  $V_{oc} = 4V$ ,  $i_{sc} = 22mA$ , SO

$$i_N = 22mA, \quad R_N = \frac{V_{oc}}{i_N} = \frac{V_{TH}}{i_N} = R_{TH} = 182\Omega$$



IT IS EASY TO CONVERT BACK AND FORTH BETWEEN NORTON & THÉVENIN EQUIVALENTS:

