

OUTLINE

LAST TIME: (CH. 5)
COMPLEX NUMBERS
MAGNITUDE
COMPLEX CONJUGATE
EULER'S FORMULA

THIS TIME: (CH. 7)

ALTERNATING CURRENT

- SINE & COSINE, ω , f , T

- POWER

- PHASE SHIFT

- COMPLEX REPRESENTATION (PHASORS)

NEXT TIME: (CH. 8)

- CAPACITORS

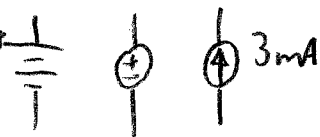
- INDUCTORS

- COMPLEX IMPEDANCE

CONFIDENTIAL

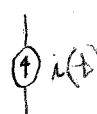
SO FAR, WE HAVE BEEN DEALING WITH "DIRECT CURRENT" (DC):

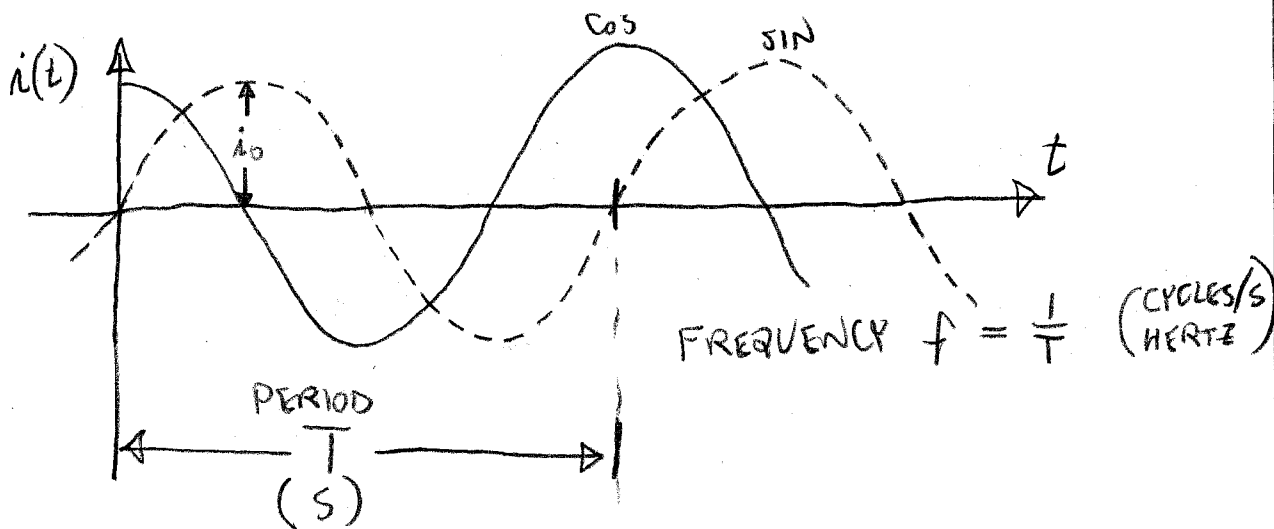
$$V = \text{CONSTANT } 5V$$

$$i = \text{CONSTANT } 3mA$$


NOW CONSIDER "ALTERNATING CURRENT" (AC). EXAMPLES:

$$i(t) = i_0 \cos(\omega t)$$

$$\text{OR } i(t) = i_0 \sin(\omega t)$$




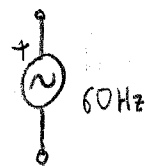
THE QUANTITY ω (RADIAN/SEC) IS CALLED THE "ANGULAR FREQUENCY". IT IS RELATED TO T & f AS FOLLOWS:

$$\omega T = 2\pi$$

$$\Rightarrow T = \frac{2\pi}{\omega} \leftarrow \text{SECONDS, } s$$

$$\Rightarrow f = \frac{\omega}{2\pi} \leftarrow \text{HERTZ, } Hz$$

WE MAY SIMILARLY HAVE
 $V(t) = V_0 \cos(\omega t)$.

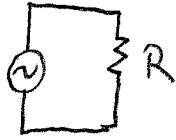
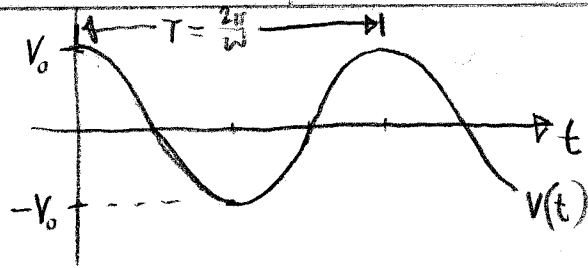


THIS IS STILL CALLED AC (ALTERNATING CURRENT), EVEN THOUGH IT IS VOLTAGE!

SAMPAD

SUPPOSE

$$V = V_0 \cos(\omega t)$$

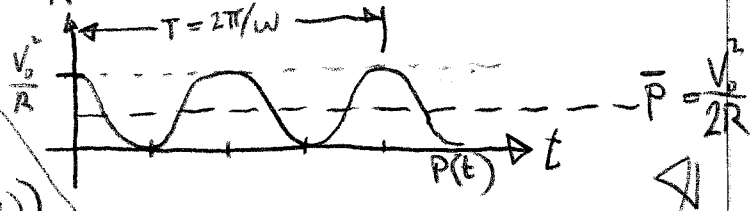


WHAT IS THE POWER DISSIPATED IN THE RESISTOR?

RECALL

$$P = iV; \quad i = \frac{V}{R} = \frac{V_0}{R} \cos(\omega t)$$

$$\begin{aligned} \therefore P &= \frac{V_0^2}{R} \cos^2(\omega t) \\ &= \frac{V_0^2}{R} \frac{1}{2} (1 + \cos(2\omega t)) \end{aligned}$$



WHAT IS THE TIME AVERAGE OF THE POWER?

$$\bar{P} = \frac{1}{T} \int_0^T P dt = \frac{1}{T} \int_0^T \frac{V_0^2}{2R} (1 + \cos(2\omega t)) dt$$

$$\text{LET } \omega t = \theta, \quad dt = \frac{d\theta}{\omega}$$

$$\bar{P} = \frac{1}{T} \int_0^{2\pi} \frac{V_0^2}{2R\omega} (1 + \cos 2\theta) d\theta = \frac{V_0^2}{2R\omega T} \left(\theta + \frac{1}{2} \sin 2\theta \right)_0^{2\pi}$$

$$= \frac{V_0^2}{2R\omega \left(\frac{2\pi}{\omega}\right)} 2\pi = \frac{1}{2} \frac{V_0^2}{R}$$

✓ COMPARE TO THE GRAPHICAL RESULT ABOVE

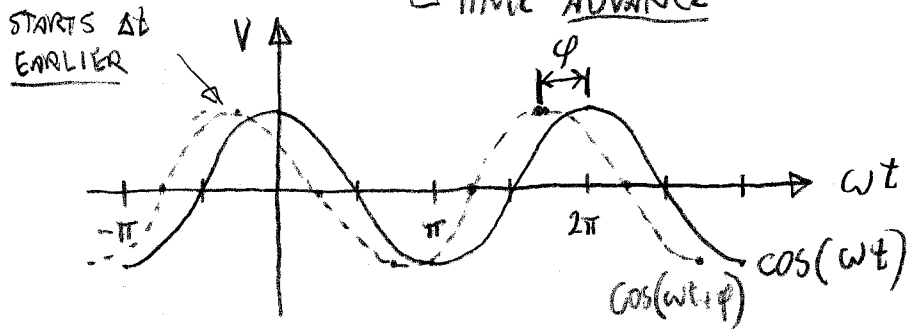
SMFAD

PHASE SHIFTS

$$V = V_0 \cos(\omega t + \phi)$$

$$= V_0 \cos(\omega [t + \Delta t]) ; \text{ so } \omega \Delta t = \phi$$

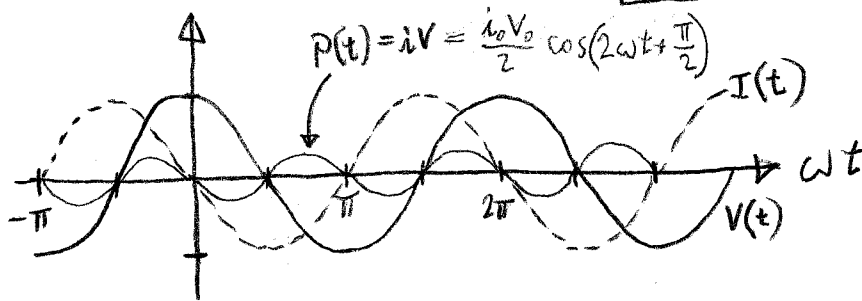
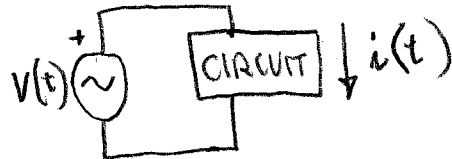
↑ TIME ADVANCE



WHAT HAPPENS IF VOLTAGE & CURRENT ARE "OUT OF PHASE" ?

i.e. $V = V_0 \cos(\omega t)$
 $i = i_0 \cos(\omega t + \phi)$ } DIFFERENT PHASE!

FOR EXAMPLE, $\phi = \frac{\pi}{2}$:



$P > 0$: CIRCUIT IS DISSIPATING POWER

$P < 0$: CIRCUIT IS A POWER SOURCE
(SENDING POWER BACK INTO THE VOLTAGE SOURCE)

Q: FOR $\phi = \frac{\pi}{2}$, WHAT IS \bar{P} ?

A: $\bar{P} = 0$. THERE IS NO AVERAGE POWER, BECAUSE THE POSITIVE PEAKS AND NEGATIVE VALLEYS EXACTLY CANCEL.

MORE GENERALLY,

$$i = i_0 \cos(\omega t + \phi)$$

$$v = v_0 \cos(\omega t)$$

THEN

$$p = i v = i_0 v_0 (\cos \omega t \cos \phi - \sin \omega t \sin \phi) \cos \omega t$$

$$= i_0 v_0 \left(\underbrace{\cos^2 \omega t}_{\frac{1}{2}(1 + \cos 2\omega t)} \cos \phi - \underbrace{\sin \omega t \cos \omega t}_{\frac{1}{2} \sin 2\omega t} \sin \phi \right)$$

$$= \frac{i_0 v_0}{2} \left[(1 + \cos 2\omega t) \cos \phi - \sin 2\omega t \sin \phi \right]$$

$$= \frac{i_0 v_0}{2} \left[\cos \phi + \underbrace{\cos 2\omega t \cos \phi - \sin 2\omega t \sin \phi}_{\cos(2\omega t + \phi)} \right]$$

$$= \frac{i_0 v_0}{2} \left[\cos \phi + \cos(2\omega t + \phi) \right]$$

↑
AMPLITUDE

↑
MEAN
VALUE

↑
TIME VARIATION
WITH MEAN OF ZERO

SIMILARLY, IF

$$i = i_0 \cos(\omega t + \phi_1), \quad \text{AND}$$

$$v = v_0 \cos(\omega t + \phi_2), \quad \text{THEN}$$

$$p = i v = \frac{i_0 v_0}{2} \left[\cos(\phi_2 - \phi_1) + \cos(2\omega t + \phi_1 + \phi_2) \right]$$

$$\therefore \bar{p} = \frac{i_0 v_0}{2} \cos(\phi_2 - \phi_1)$$

USING COMPLEX NUMBERS TO REPRESENT AC:

IF $V = V_0 \cos(\omega t + \phi)$, WE WRITE

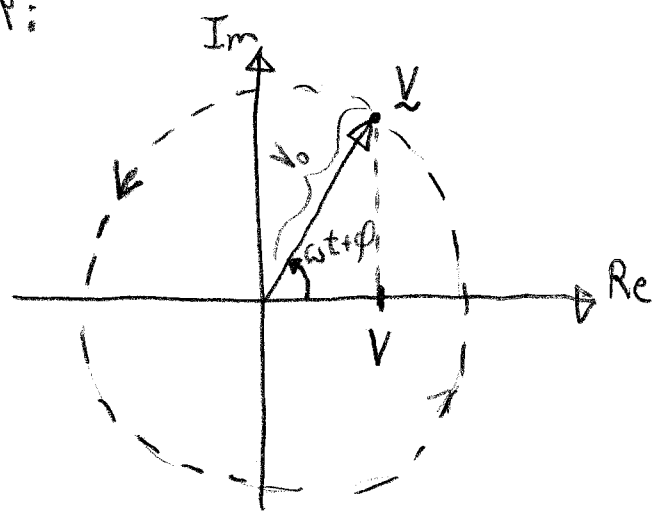
$$\tilde{V} = V_0 e^{j(\omega t + \phi)} = \underbrace{V_0}_{\substack{\text{COMPLEX} \\ \text{AMPLITUDE}}} e^{j\omega t}$$

\uparrow AMPLITUDE \uparrow ANGULAR FREQUENCY \uparrow PHASE SHIFT

$$\tilde{V}_0 = V_0 e^{j\phi}$$

THE COMPLEX AMPLITUDE ENCODES THE PHASE SHIFT, AS WELL AS THE AMPLITUDE.

GRAPHICALLY:



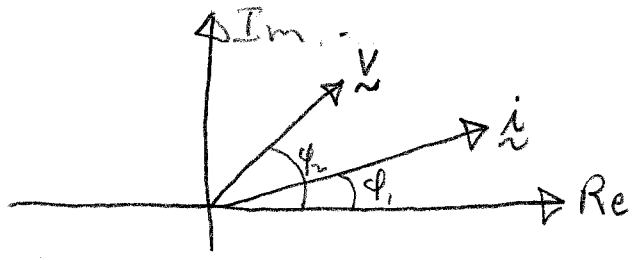
THE "PHASOR" (ARROW) HOLDS CONSTANT RADIUS V_0 AS IT ROTATES COUNTER CLOCKWISE.

RECALL THAT IN OUR STUDY OF POWER, WE FOUND

IF $i = i_0 \cos(\omega t + \phi_1)$, $V = V_0 \cos(\omega t + \phi_2)$, THEN

$$\bar{P} = \frac{1}{2} i_0 V_0 \cos(\phi_2 - \phi_1) = \frac{1}{2} i_0 V_0 \cos(\phi_1 - \phi_2)$$

THIS IS LIKE A DOT PRODUCT, $\vec{i} \cdot \vec{V}$.



CONSIDER: $\vec{z} \cdot \vec{z} = |\vec{z}|^2 \equiv \vec{z}^* \cdot \vec{z}$

BY ANALOGY, TRY $\bar{P} \equiv \frac{1}{2} \vec{i} \cdot \vec{V} = \frac{1}{2} \vec{i}^* \cdot \vec{V} = \frac{1}{2} i_0 e^{-j(\omega t + \phi_1)} v_0 e^{j(\omega t + \phi_2)}$

$$= \frac{1}{2} i_0 V_0 e^{j(\phi_2 - \phi_1)}$$

Q: IS THIS RIGHT?

SMPAL

A: ALMOST.

$$\bar{P} = \frac{1}{2} \operatorname{Re}(i^* v) = \frac{1}{2} \operatorname{Re}(i v^*)$$

$$= \frac{1}{2} i_0 v_0 \cos(\phi_2 - \phi_1) = \frac{1}{2} i_0 v_0 \cos(\phi_1 - \phi_2)$$

TO SEE HOW THIS "DOT" PRODUCT WORKS, CONSIDER
THE ORDINARY DOT PRODUCT:

$$\begin{bmatrix} a \\ b \end{bmatrix} \cdot \begin{bmatrix} c \\ d \end{bmatrix} = ac + bd$$

IN THE COMPLEX PLANE,

$$(a + jb) \cdot (c + jd) \equiv \operatorname{Re}[(a + jb)^* (c + jd)]$$

$$= \operatorname{Re}[(a - jb)(c + jd)] =$$

$$= \operatorname{Re}[ac + bd + j(ad - bc)]$$

$$= ac + bd \quad \checkmark$$

INTERESTINGLY, THE IMAGINARY PIECE
IS THE CROSS PRODUCT!