

SO FAR:

- COMPLEX NUMBERS (CH. 5)
- ALTERNATING CURRENT (CH. 7)

TODAY:

CAPACITORS, INDUCTORS, AND IMPEDANCE (CH. 8)

NEXT TIME:

AC CIRCUIT ANALYSIS AND RESONANCE (CHS. 9-10)

PLEASE UNDERSTAND THAT MY DESCRIPTION OF THE PHYSICS WILL BE SUPERFICIAL. YOU WILL GET A MUCH MORE PHYSICALLY SATISFYING INTRO TO CAPACITANCE AND INDUCTANCE IN PH5X 242/222. IN THIS CLASS, HOWEVER, WE'LL LEARN SOME ADVANCED TECHNIQUES FOR SOLVING CIRCUITS WITH ANY COMBINATION OF RESISTORS, CAPACITORS AND INDUCTORS.

VISUAL AID: VARIOUS CAPACITORS

CAPACITORS

# CAPACITORS

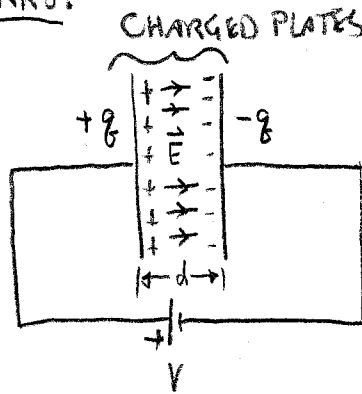
A CAPACITOR IS A DEVICE FOR STORING ELECTRIC CHARGE. THE CAPACITOR IS VERY DIFFERENT FROM A BATTERY, IN WHICH A NEARLY CONSTANT VOLTAGE IS SUPPLIED BY AN ELECTROCHEMICAL REACTION. THE CAPACITOR HAS A VOLTAGE DIRECTLY PROPORTIONAL TO ITS CHARGE:

$$Q = CV \quad (1)$$

CHARGE →  $Q$  ← VOLTAGE

← CAPACITANCE  $C$

## HOW IT WORKS:



THE ELECTRIC FIELD DUE TO THE CHARGES CORRESPONDS TO THE VOLTAGE!

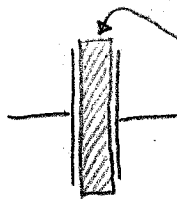
$$V = \int \vec{E} \cdot d\vec{l} = Ed, \text{ WHERE}$$

$$E = \frac{Q}{\epsilon_0 A} \leftarrow \text{PLATE AREA}$$

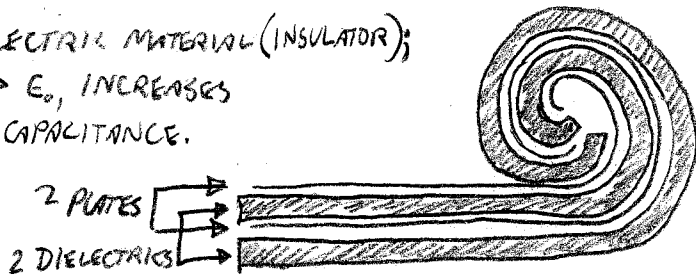
$$\text{SO } V = \left(\frac{d}{\epsilon_0 A}\right) Q, \text{ SO } C = \frac{\epsilon_0 A}{d}$$

THIS WILL NOT MAKE MUCH SENSE UNLESS YOU HAVE HAD 242 OR 222.

THIS "PARALLEL PLATE" CAPACITOR IS THE SIMPLEST SORT. MODERN CAPS RELY ON ANY OR ALL OF THE FOLLOWING INNOVATIONS:

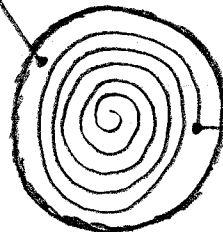


DIELECTRIC MATERIAL (INSULATOR);  $\epsilon \gg \epsilon_0$ , INCREASES THE CAPACITANCE.



FOLDED OR ROLLED TOGETHER FOR COMPACTNESS

CONDUCTIVE ELECTROLYTE SEAVES AS THE OTHER "PLATE"

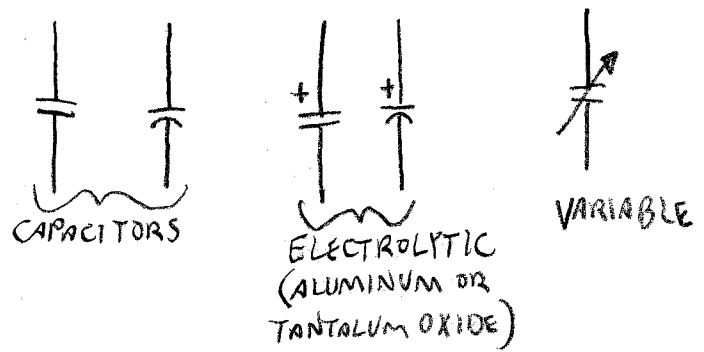


METALLIC FILM COVERED WITH THIN OXIDE LAYER

ELECTROLYTIC CAPS HAVE LARGE C, BUT TEND TO EXPLODE WHEN CHARGED "BACKWARDS"!

STRAIGHT

### CIRCUIT SYMBOLS:



### UNITS:

$$C = \frac{Q}{V} = \text{COUL/VOLT} = \underline{\text{FARAD}} \text{ (F)}$$

### TYPICAL VALUES: \*

TYPE	RANGE
MYLAR, POLYESTER	1000 pF - 10 μF
POLYPROPYLENE	2 pF - 0.5 μF
ELECTROLYTIC (Al, Ta)	1 μF - 1 F

1 F IS A LOT OF CAPACITANCE!

ENERGY STORAGE: HOW MUCH WORK TO CHARGE TO VOLTAGE V?

V = ENERGY PER CHARGE

$$\int dW = \int V dq = \int \frac{q}{C} dq$$

$$W = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} CV^2$$

\* FROM FORTNEY'S PRINCIPLES OF ELECTRONICS, TABLE 2.1

WITH RESISTORS, WE USE OHM'S LAW TO ANALYZE CIRCUITS:

$$V = iR$$

WE CAN DO SOMETHING SIMILAR WITH CAPACITORS:

$$\int i dt = q = CV$$

TAKE TIME DERIVATIVE:

$$\frac{dq}{dt} = i = C \frac{dV}{dt}$$

NOW, SUPPOSE WE HAVE AN AC VOLTAGE:

$$\underline{V} = \underline{V}_0 e^{j\omega t} = \underline{V}_0 e^{j(\omega t + \phi)}$$

$$\underline{i} = C \frac{d\underline{V}}{dt} = C(j\omega) \underline{V}_0 e^{j\omega t} = j\omega C \underline{V}$$

$$\therefore \underline{V} = \left( \frac{1}{j\omega C} \right) \underline{i}$$

THIS QUANTITY, WITH UNITS OF  $\Omega$ , IS CALLED CAPACITIVE IMPEDANCE.

$$Z_c = \frac{1}{j\omega C} = \frac{-j}{\omega C}$$

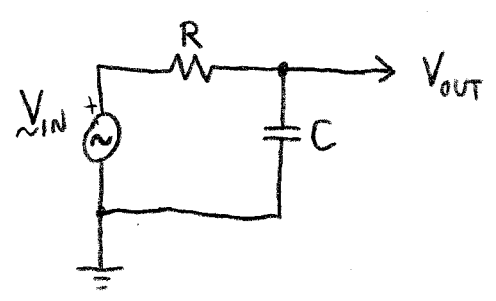
WITH AC CIRCUITS, CAPACITORS CAN BE TREATED JUST LIKE RESISTORS USING  $Z_c$  AS THE "RESISTANCE" OF THE CAPACITOR.

ALL OUR USUAL TRICKS FOR D.C. CIRCUIT ANALYSIS CAN THEN BE USED TO ANALYZE A.C. CIRCUITS WITH ANY COMBINATION OF RESISTORS & CAPACITORS, FOR EXAMPLE:

- SERIES & PARALLEL
- VOLTAGE DIVIDER FORMULA
- NODE VOLTAGE ANALYSIS
- KIRCHOFF'S LAWS (KCL, KVL)

# EXAMPLE

## LOWPASS FILTER



WE COULD JUST USE THE VOLTAGE DIVIDER FORMULA:

$$\underline{V}_{OUT} = \frac{R_2 \underline{V}_{IN}}{R_1 + R_2} = \frac{Z_C \underline{V}_{IN}}{Z_C + R}$$

RECALL  $Z_C = 1/j\omega C$ .

$$\therefore \underline{V}_{OUT} = \left( \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R} \right) \underline{V}_{IN} = \left( \frac{1}{1 + j\omega RC} \right) \underline{V}_{IN}$$

$$\underline{V}_{OUT} = \left( \frac{1 - j\omega RC}{1 + \omega^2 R^2 C^2} \right) \underline{V}_{IN}$$

WE'LL INTERPRET THIS RESULT MORE THOROUGHLY NEXT WEEK, BUT FOR NOW WE CAN SEE THAT:

- WE GOT  $V_{OUT}$  IN TERMS OF  $V_{IN}$  USING OHM'S LAW!
- THE RESULT DEPENDS ON FREQUENCY ( $f = \omega/2\pi$ ).
- THE PHYSICAL PART IS THE REAL PART. IF

$$\underline{V}_{IN} = V_0 e^{j\omega t}, \quad V_0 \in \mathbb{R}, \quad \text{THEN } V_{IN} = \text{Re}(\underline{V}_{IN}) = \boxed{V_0 \cos \omega t}$$

$$\underline{V}_{OUT} = \frac{1 - j\omega RC}{1 + \omega^2 R^2 C^2} V_0 [\cos \omega t + j \sin \omega t]$$

$$\underline{V}_{OUT} = \text{Re}(\underline{V}_{OUT}) = \boxed{V_0 [\cos \omega t + \omega RC \sin \omega t] / [1 + \omega^2 R^2 C^2]}$$

AN INDUCTOR IS SIMPLY A DEVICE THAT OPPOSES ANY CHANGE IN CURRENT. ITS BEHAVIOR IS DESCRIBED BY

$$V = +L \frac{di}{dt}$$

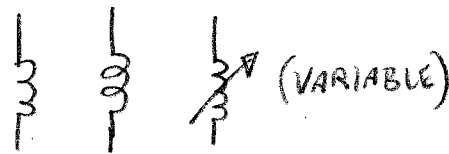
VOLTAGE OPPOSES  $\frac{di}{dt}$

$\frac{L}{i}$

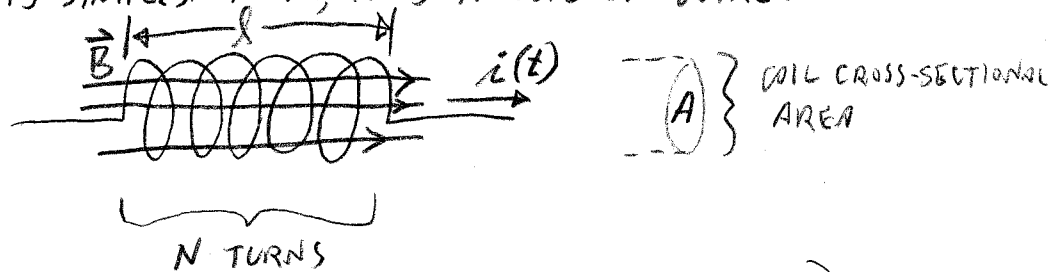
WHERE  $L$ , THE INDUCTANCE, IS MEASURED IN HENRIES:

$$\text{UNITS: } L = \frac{V}{\left(\frac{di}{dt}\right)} = \frac{V \cdot s}{A} = \text{HENRY (H)}$$

CIRCUIT SYMBOLS:



WHAT, PHYSICALLY, IS AN INDUCTOR, AND WHY DOES IT BEHAVE THIS WAY? IN ITS SIMPLEST FORM, IT IS A COIL OF WIRE:



ACCORDING TO AMPERE'S LAW, THE MAGNETIC FIELD IS

$$B = \mu_0 N i / l$$

FARADAY'S LAW SAYS THERE IS A VOLTAGE

$$V = +AN \frac{dB}{dt}$$

SO,

$$V = + \left( \frac{\mu_0 N^2 A}{l} \right) \frac{di}{dt}$$

$\uparrow$   
 $L$

THIS PART WILL NOT MAKE MUCH SENSE UNLESS YOU'VE HAD 242 OR 222.

SOME INDUCTORS ARE WOUND ON CORES WITH  $\mu \gg \mu_0$ , WHICH ENHANCES THE INDUCTANCE.

# ANALYSIS OF THE INDUCTOR IN AN A.C. CIRCUIT:

SUPPOSE  $\underline{V} = V_0 e^{j\omega t}$   
 ASSUME  $\underline{i} = i_0 e^{j\omega t}$

NOTE THAT COMPLEX AMPLITUDES ALLOW THE PHASE OF  $V$  AND  $i$  TO DIFFER. ONLY THE FREQUENCIES ARE ASSUMED TO BE THE SAME.

TAKING

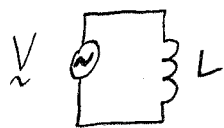
$$\underline{V} = L \frac{d\underline{i}}{dt}$$

$$\Rightarrow \underline{V} = L \frac{d(i_0 e^{j\omega t})}{dt} = j\omega L \underline{i}$$

WE AGAIN HAVE AN "OHM'S LAW",

$$\underline{V} = Z_L \underline{i}, \quad Z_L = j\omega L$$

## SIMPLE EXAMPLE



PLOT  $V(t)$  AND  $i(t)$

THE RELATIONSHIP BETWEEN VOLTAGE AND CURRENT FOR THE INDUCTOR IS:

$$\underline{i} = \underline{V} / Z_L = \underline{V} / (j\omega L) = \frac{V_0 e^{j\omega t}}{j\omega L}$$

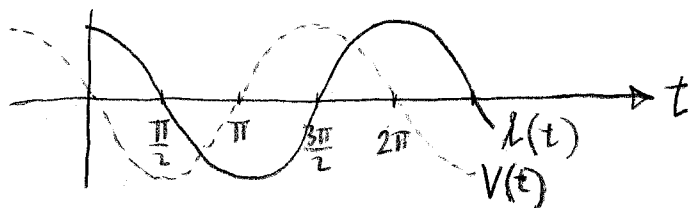
NOW,  $\frac{1}{j} = -j$ , SO  $\underline{i} = \frac{-j}{\omega L} \underline{V}$ .

LET'S PUT IT IN POLAR FORM:

PHASE:  $e^{j\varphi} = e^{-j\pi/2}$   
 $\Rightarrow \varphi = -\pi/2$

$$\underline{i} = \frac{1}{\omega L} e^{j\pi/2} \underline{V}$$

SO IN GENERAL, CURRENT LAGS VOLTAGE BY  $\frac{\pi}{2}$  ( $90^\circ$ ) IN INDUCTORS.\*



\* WHAT WOULD THE CORRESPONDING LAW BE FOR CAPACITORS?