

LAST TIME (CH. 8)

- CAPACITORS
- INDUCTORS
- IMPEDANCE

TODAY (CHS. 9-10)

- IMPEDANCE SUMMARY
- TRANSFER FUNCTION (GAIN AND PHASE)
- EXAMPLES
 - HIGH PASS FILTER (RC)
 - RESONANCE (LC)

IMPAD

ANNOUNCEMENT:

LAST TIME, I GOT THE SIGN WRONG IN THE INDUCTANCE EQUATION:

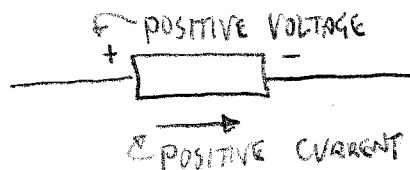
$$V = L \frac{di}{dt} \leftarrow \text{CORRECT}$$

FAISSLER HAS IT RIGHT. I HAVE FIXED MY LECTURE NOTES ACCORDINGLY.

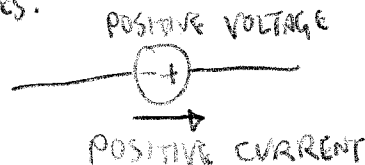
ACTUALLY, I WAS ONLY FOLLOWING MY OWN UNDERGRADUATE TEXTBOOK! MOST PHYSICS (NOT ELECTRONICS) TEXTS HAVE A MINUS SIGN IN THE EQUATION. IT HAS TO DO WITH SIGN CONVENTIONS.

HERE IS THE SIGN CONVENTION WE USE.

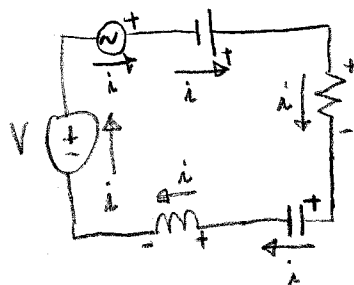
PASSIVE COMPONENTS: (L, R, C)



POWER SOURCES:



EXAMPLES:



IMPEDANCE SUMMARY

INDUCTORS, RESISTORS & CAPACITORS CAN BE MODELED USING IMPEDANCE

COMPONENT	VALUE	UNITS	DEFINING EQUATION	IMPEDANCE, Z (Ω)
RESISTOR	R	Ω	V = iR	R
CAPACITOR	C	F	Z = CV (i = C dV/dt)	1/jωC
INDUCTOR	L	H	V = L di/dt	jωL

$$\underline{v} = \underline{v}_0 e^{j\omega t}, \quad \underline{i} = \underline{i}_0 e^{j\omega t}$$

$$v = \text{Re}(\underline{v}), \quad i = \text{Re}(\underline{i})$$

OHM'S LAW GENERALIZES TO:

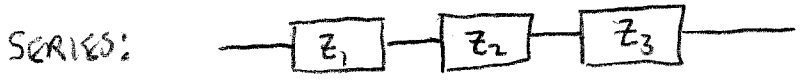
$$\underline{v} = \underline{i} Z$$

FREQUENCY:

$$f = \frac{\omega}{2\pi} = \frac{1}{T}$$

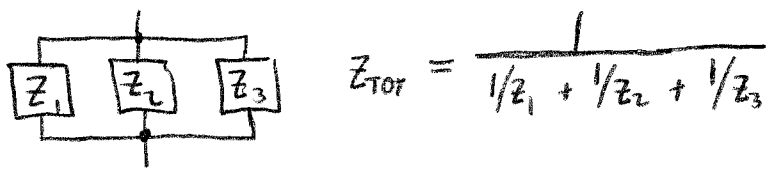
UNITS: HZ (CYCLES PER SECOND)

NOTE: IMPEDANCES ADD JUST LIKE RESISTANCE



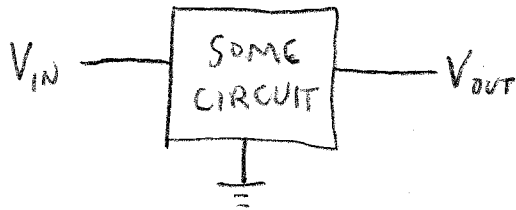
$$Z_{TOT} = Z_1 + Z_2 + Z_3$$

PARALLEL:



TRANSFER FUNCTIONS *

SUPPOSE SOME CIRCUIT MODIFIES AN INPUT VOLTAGE:

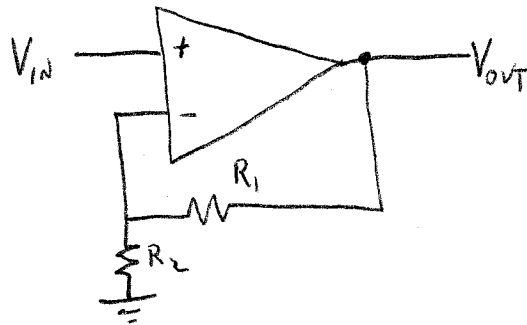


FURTHER SUPPOSE:

$$V_{OUT} = G V_{IN}$$

↑
TRANSFER FUNCTION

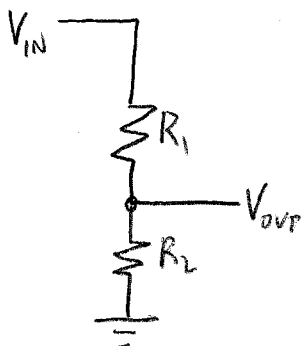
EXAMPLE:



$$V_{OUT} = \frac{R_1 + R_2}{R_2} V_{IN}$$

$$G = \frac{R_1 + R_2}{R_2}$$

EXAMPLE:



$$V_{OUT} = \frac{R_2}{R_1 + R_2} V_{IN}$$

$$G = \frac{R_2}{R_1 + R_2}$$

* YOUR TEXT SIMPLY CALLS G THE GAIN.

MORE GENERALIZED NOTION OF A TRANSFER FUNCTION:

LET $\tilde{V}_{IN} = \tilde{V}_0 e^{i\omega t}$

ASSUME $\tilde{V}_{OUT} = \tilde{G} \tilde{V}_{IN}$

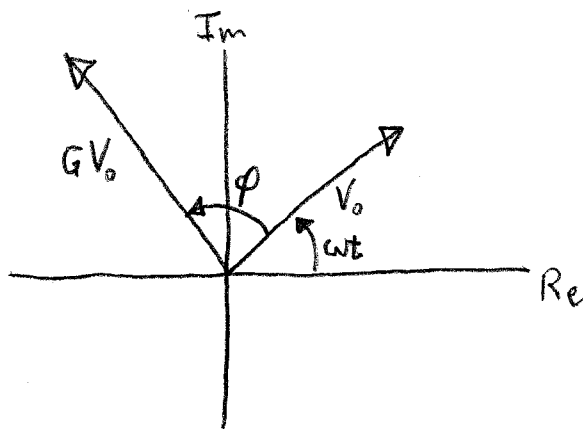
NOW, IF $\tilde{G} \in \mathbb{C}$, THEN IT HAS BOTH A MAGNITUDE AND A PHASE:

LET $G \equiv |\tilde{G}|$. THIS IS THE GAIN. *

$\tilde{G}/G = e^{i\phi}$. THIS IS THE PHASE FACTOR.

SO $\tilde{V}_{OUT} = G e^{i\phi} \tilde{V}_{IN} = G \tilde{V}_0 e^{i(\omega t + \phi)}$

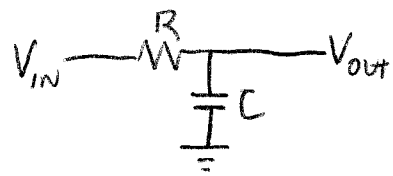
PICTORALLY:
(PHASORS)



TYPICALLY, G AND ϕ TURN OUT TO BE FUNCTIONS OF ω . THIS IS WHY WE CALL $\tilde{G} = \tilde{G}(\omega)$ A TRANSFER FUNCTION.

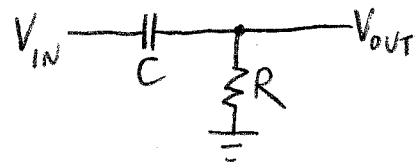
* FAISSLER CALLS \tilde{G} THE GAIN. I THINK MY TERMINOLOGY (\tilde{G} = TRANSFER FXN, $|\tilde{G}|$ = GAIN) IS A LITTLE MORE STANDARD, BUT I'LL TRY TO ALWAYS BE CLEAR ABOUT WHAT I'M ASKING FOR.

LAST TIME, WE ANALYZED AN RC LOWPASS FILTER:



$$V_{OUT} = \underbrace{\left(\frac{1 - j\omega RC}{1 + \omega^2 R^2 C^2} \right)}_{G(\omega)} V_{IN}$$

HERE IS A HIGH PASS FILTER:



USING NODE VOLTAGE ANALYSIS:

$$\frac{V_{IN} - V_{OUT}}{Z_C} + \frac{0 - V_{OUT}}{R} = 0 \Rightarrow R(V_{IN} - V_{OUT}) - Z_C V_{OUT} = 0$$

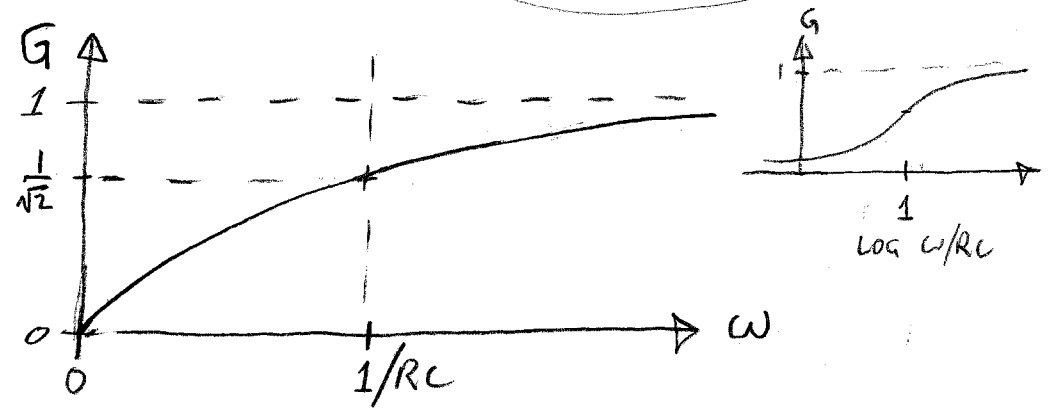
$$\Rightarrow V_{OUT} = \left(\frac{R}{R + Z_C} \right) V_{IN}$$

$$\begin{aligned} \text{SO } G(\omega) &= \frac{R}{Z_C + R} = \frac{R}{\frac{1}{j\omega C} + R} = \frac{j\omega RC (1 - j\omega RC)}{(1 + j\omega RC)(1 - j\omega RC)} \\ &= \frac{\omega^2 R^2 C^2 + j\omega RC}{1 + \omega^2 R^2 C^2} \end{aligned}$$

WHAT IS THE GAIN?

$$G = \left| \frac{\omega^2 R^2 C^2 + j\omega RC}{1 + \omega^2 R^2 C^2} \right| = \frac{\sqrt{\omega^4 R^4 C^4 + \omega^2 R^2 C^2}}{1 + \omega^2 R^2 C^2}$$

$$= \omega RC \frac{\sqrt{1 + \omega^2 R^2 C^2}}{1 + \omega^2 R^2 C^2} = \frac{\omega RC}{\sqrt{1 + (\omega RC)^2}}$$



WHAT ABOUT THE PHASE?

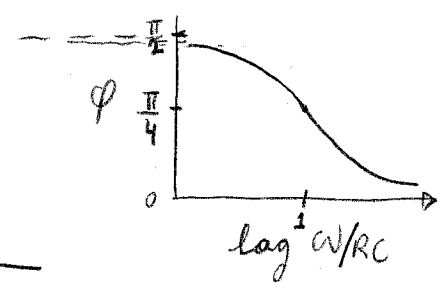
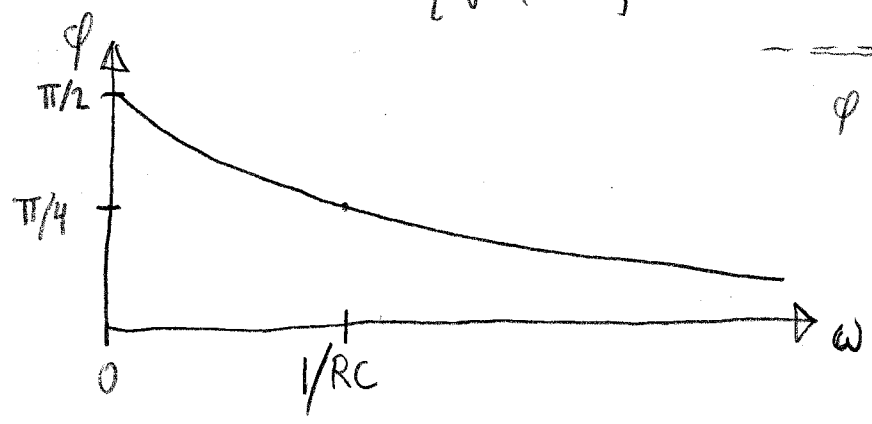
$$e^{i\phi} = \underline{G}/G = \frac{\omega^2 R^2 C^2 + j\omega RC}{1 + (\omega RC)^2} \cdot \frac{\sqrt{1 + (\omega RC)^2}}{\omega RC}$$

$$= \frac{\omega RC + j}{\sqrt{1 + (\omega RC)^2}} = \cos \phi + j \sin \phi$$

SO, $\cos \phi = \frac{\omega RC}{\sqrt{1 + (\omega RC)^2}}$, $\sin \phi = \frac{1}{\sqrt{1 + (\omega RC)^2}}$

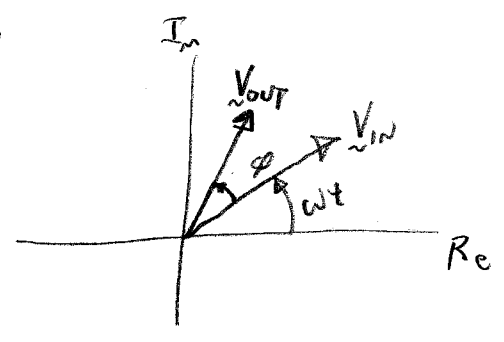
NOTE THAT $\sin \phi > 0$ AND $\cos \phi > 0$, SO THE PHASE ϕ IS IN QUADRANT I ($0 \leq \phi \leq \frac{\pi}{2}$).

$$\phi = \sin^{-1} \left[\frac{1}{\sqrt{1 + (\omega RC)^2}} \right]$$

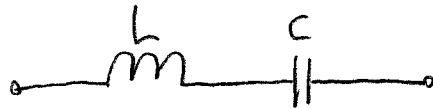


Q: $\phi \geq 0$ MEANS WHAT?

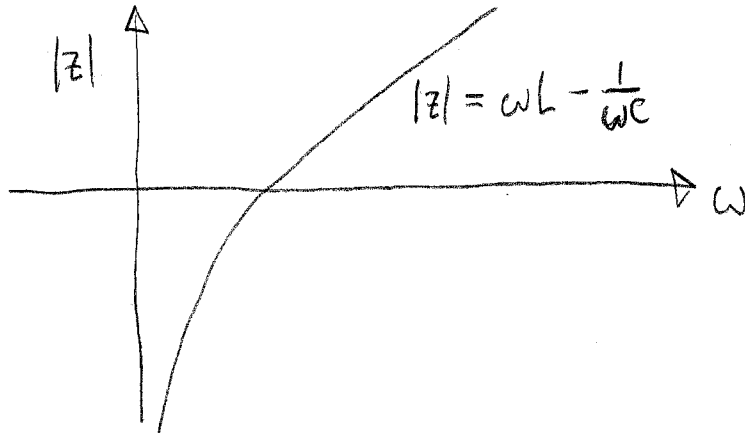
A: V_{OUT} LEADS V_{IN} .



WHAT IS THE IMPEDANCE OF THIS COMBINATION?



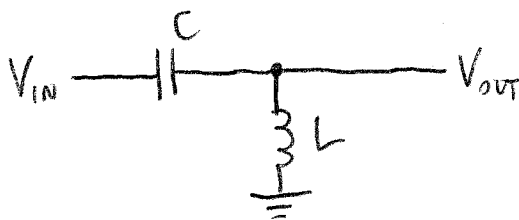
$$Z = Z_L + Z_C = j\omega L + \frac{1}{j\omega C} = j \left[\omega L - \frac{1}{\omega C} \right]$$



NOTE THAT $Z=0$ WHEN $\omega L = \frac{1}{\omega C}$, I.E. $\omega = \frac{1}{\sqrt{LC}}$.

THIS CONDITION IS CALLED RESONANCE.

INVESTIGATING FURTHER, LET'S APPLY AN A.C. VOLTAGE AND CONSIDER THE RESULTING VOLTAGE ACROSS THE INDUCTOR:

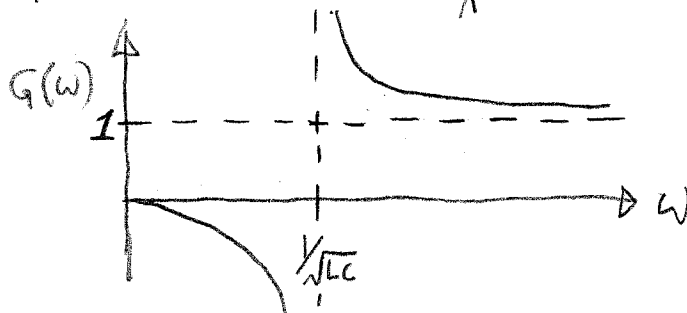


$$V_{OUT} = \left(\frac{Z_L}{Z_L + Z_C} \right) V_{IN}$$

$$\underline{G}(\omega) = \frac{j\omega L}{j\omega L + \frac{1}{j\omega C}} = \frac{-\omega^2 LC}{-\omega^2 LC + 1} = \frac{\omega^2 LC}{\omega^2 LC - 1} \in \mathbb{R}$$

SO $\underline{G}(\omega) = G(\omega)$; $e^{i\phi} = 1$; $\phi = 0$

THE VOLTAGE ACROSS THE INDUCTOR V_{OUT} IS IN PHASE WITH V_{IN} .



$G \rightarrow 1$ AS $\omega \rightarrow \infty$
 $G < 1$ FOR $\omega < \frac{1}{\sqrt{LC}}$
 $G > 1$ FOR $\omega > \frac{1}{\sqrt{LC}}$

WHAT HAPPENS IN THE NEIGHBORHOOD OF $\omega \approx \frac{1}{\sqrt{LC}}$?

ω	G
$\frac{1}{\sqrt{LC}} + \epsilon$	$G \gg 1$ $G \rightarrow \infty$
$\frac{1}{\sqrt{LC}}$	UNDEFINED! $ i \rightarrow \infty, V_{out} \rightarrow \infty$
$\frac{1}{\sqrt{LC}} - \epsilon$	$G \ll -1$ $G \rightarrow -\infty$

IN REAL LIFE, IF YOU TRY TO TUNE TO RESONANCE (ASSUMING NOTHING EXPLODES!), THEN YOU LEARN THAT THE CIRCUIT HAS A LITTLE RESISTANCE. THIS ELIMINATES THE BAD MATHEMATICAL BEHAVIOR. IT ALSO INTRODUCES A GRADUAL PHASE SHIFT IN THE NEIGHBORHOOD OF RESONANCE.