

WE HAVE COVERED A SIGNIFICANT AMOUNT OF MATERIAL IN 13 LECTURES ON DC AND AC CIRCUITS:

- OHM'S AND KIRCHOFF'S LAWS
- CIRCUIT ANALYSIS TECHNIQUES
- DIODES & TRANSISTORS
- OPERATIONAL AMPLIFIERS
- NEGATIVE AND POSITIVE FEEDBACK  
(AMPLIFIERS, COMPARATORS, SCHMITT TRIGGERS)
- COMPLEX NUMBERS, PHASE, AND ALTERNATING CURRENT
- CAPACITORS, INDUCTORS, AND IMPEDANCE
- ANALYSIS OF SIMPLE AC CIRCUITS (GAIN AND PHASE)

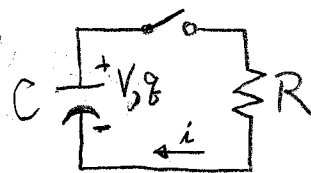
TODAY, WE'LL FINISH BY DISCUSSING TRANSIENT ANALYSIS (CH.12)

- RC CIRCUITS
- LR CIRCUITS

# RC CIRCUITS

## DISCHARGING A CAPACITOR

SUPPOSE THE CAPACITOR IS INITIALLY CHARGED TO VOLTAGE



$V_0$ . AT TIME  $t=0$ , THE SWITCH IS CLOSED (—●—●—), COMPLETING THE CIRCUIT. FIND  $q(t)$ ,  $V(t)$ , AND  $i(t)$ .

THE CAPACITOR IS GOVERNED BY ITS DEFINING EQUATION,

$$q = CV.$$

NOW, APPLY KVL:

$$\frac{q}{C} - iR = 0 \quad (1)$$

BUT  $q$  AND  $i$  ARE JUST DIFFERENT WAYS OF KEEPING TRACK OF THE SAME THING:

$$i = -\frac{dq}{dt}, \quad (\text{WHY MINUS? LOOK AT THE CIRCUIT DIAGRAM})$$

SO EQUATION (1) BECOMES:

$$\frac{q}{C} + R \frac{dq}{dt} = 0$$

THIS IS A DIFFERENTIAL EQUATION, BUT A PARTICULARLY EASY ONE:

$$\frac{dq}{dt} = -\frac{q}{RC} \Rightarrow \int \frac{dq}{q} = \int \frac{-dt}{RC}$$

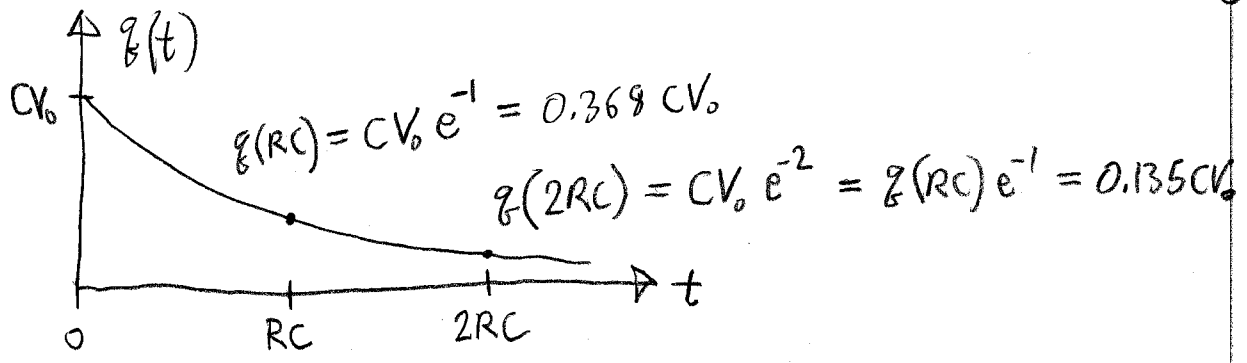
$$\Rightarrow \ln q = -\frac{t}{RC} + a$$

$$q(t) = e^{(-t/RC + a)} = e^a e^{-t/RC} = A e^{-t/RC}$$

Q: HOW DO WE FIND THE ARBITRARY CONSTANT,  $A$ ?

A: AT  $t=0$ ,  $V=V_0$ , SO  $q(0) = CV_0 = A$ .

$$\therefore \boxed{q(t) = CV_0 e^{-t/RC}} \Rightarrow \left. \begin{aligned} q(0) &= CV_0 \\ q(\infty) &= 0 \end{aligned} \right\} \text{MAKES SENSE!}$$



HOW ABOUT  $V(t)$ ?

$V = \frac{q}{C}$ , SO  $V = V_0 e^{-t/RC}$  — SAME BEHAVIOR AS THE CHARGE,

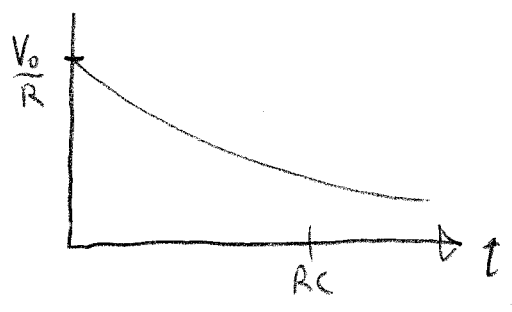
AND NOW THE CURRENT; RECALL

$$i(t) = - \frac{dq}{dt}$$

$$= - CV_0 \left( \frac{-1}{RC} \right) e^{-t/RC}$$

$$i(t) = \frac{V_0}{R} e^{-t/RC}$$

SO  $i(0) = \frac{V_0}{R}$  ✓ MAKES SENSE!



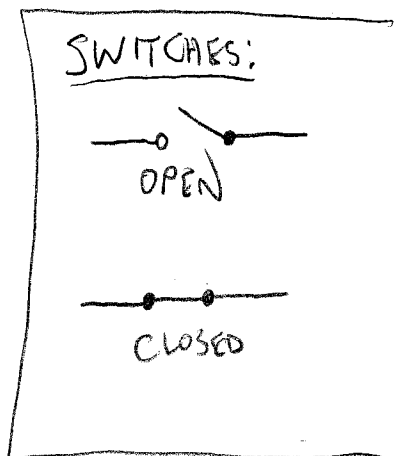
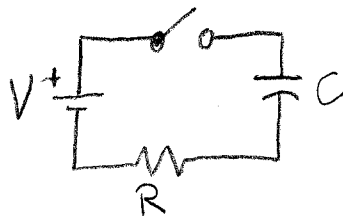
NOTE THE DIMENSIONLESS VARIABLES:

$t \rightarrow t/RC$ ,  $V \rightarrow V/V_0$ ,  $q \rightarrow q/CV_0$ ,  $i \rightarrow iR/V_0$

$$\frac{V}{V_0} = \frac{q}{CV_0} = \frac{iR}{V_0} = e^{-t/RC}$$

# CHARGING A CAPACITOR

SUPPOSE THE CAPACITOR IS INITIALLY UNCHARGED ( $q=0$ ). CLOSE THE SWITCH AT TIME  $t=0$ .



RECALL:  $q = CV$

KVL:  $V - \frac{q}{C} - iR = 0$  (2)

NOW,  $i = \frac{dq}{dt}$  (WHY PLUS?)

SO OUR EQUATION (2) BECOMES

$CV - q - RC \frac{dq}{dt} = 0$  (3)

SUPPOSE WE TRY SOMETHING LIKE THE SOLUTION TO THE LAST PROBLEM:

PLUG INTO (3):  $q = Ae^{-t/RC}$

$CV - Ae^{-t/RC} + Ae^{-t/RC} = 0$   
 $CV = 0$  ?!

WHAT MIGHT WE DO TO FIX UP THE SOLUTION?

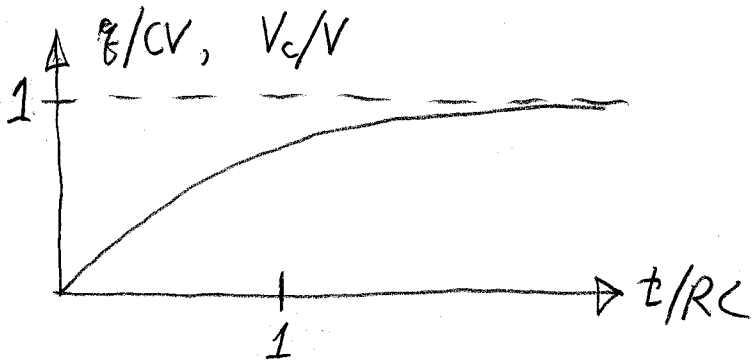
PHYSICALLY, AS  $t \rightarrow \infty$ , WE SHOULD HAVE

$\lim_{t \rightarrow \infty} q(t) \rightarrow CV$  (WHY?).

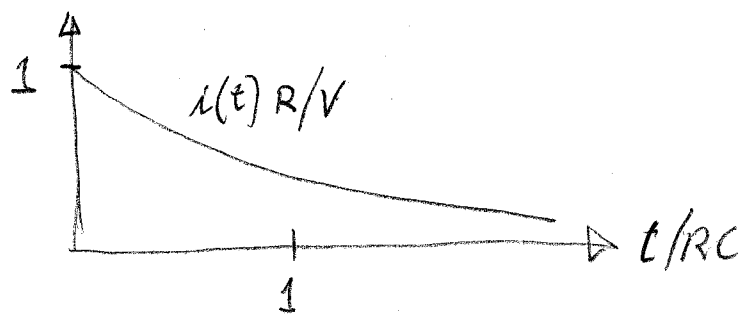
TRY  $q(t) = Ae^{-t/RC} + CV$  ✓ THIS SATISFIES (3).

NOW, WE WANT  $q(0) = 0$ , SO  $A = -CV$ .

$\therefore q(t) = CV(e^{-t/RC} - 1)$



BUT  $i(t) = \frac{V}{R} e^{-t/RC}$        $\left( \frac{iR}{V} = e^{-t/RC} \right)$



GENERAL CONCEPT FOR TRANSIENT ANALYSIS:

GIVEN A TIME-VARYING QUANTITY  $x(t)$  (PERHAPS  $i, q, V$ ),

- ① FIGURE OUT THE INITIAL VALUE,  
 $x(0) = X_0$
- ② FIGURE OUT THE FINAL VALUE,  
 $\lim_{t \rightarrow \infty} x(t) = X_{\infty}$
- ③ FIND THE RELEVANT TIME CONSTANT  $\tau$   
(FOR RC CIRCUITS,  $\tau = RC$ )

④ THE SOLUTION IS:

$$x(t) = [X_0 - X_{\infty}] e^{-t/\tau} + X_{\infty}$$

} WORKS FOR RC OR LR. LC OR LRC CAN OSCILLATE.

⑤ USE KVL TO FIND THE DIFFERENTIAL EQUATION. PLUG IN AND CHECK YOUR SOLUTION.

# RL CIRCUITS

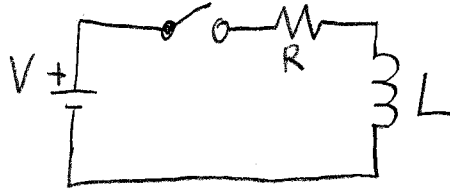
$$L \left\{ \begin{array}{l} \uparrow i(t) \\ \end{array} \right\} R \quad V_L = L \frac{di}{dt}$$

$$\text{KVL: } -L \frac{di}{dt} - iR = 0$$

$$\frac{di}{dt} = -\frac{R}{L} i \Rightarrow i = A e^{-tR/L}$$

SO, THE TIME CONSTANT FOR AN RL CIRCUIT IS  $\tau = \frac{L}{R}$

EXAMPLE



SWITCH IS CLOSED AT  $t=0$ .  
FIND  $i(t)$ .

①  $i(0) = 0$

②  $i(\infty) = V/R$  (DISCUSS?)

③  $\tau = L/R$

④  $i(t) = \left[ 0 - \frac{V}{R} \right] e^{-Rt/L} + \frac{V}{R} = \frac{V}{R} (1 - e^{-Rt/L})$

⑤ KVL:  $V - iR - L \frac{di}{dt} = 0$ ; PLUG IN MY SOLUTION:

$$V - V(1 - e^{-Rt/L}) - \cancel{L} \left( \frac{-V}{R} \right) \left( \frac{-R}{L} \right) e^{-Rt/L} = 0?$$

$$+ V e^{-Rt/L} - V e^{-Rt/L} = 0 \quad \checkmark$$

Q: WHAT ABOUT THE VOLTAGE ACROSS THE INDUCTOR,  $V_L(t)$ ?

A:  $V_L = L \frac{di}{dt} = \frac{LV}{R} \cdot \frac{R}{L} \cdot e^{-Rt/L} = V e^{-Rt/L}$

EXERCISE: TRY DERIVING  $V_L(t)$  FROM SCRATCH USING THE 5-STEP PROCESS.